A Framework on Wideband Sensing and Direction Finding for Location Aware Public Safety Cognitive Radio

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Abstract—Beside providing solutions to the interoperability problem of the first responders, location awareness feature of the cognitive radios can be beneficial for victim and first responder location estimation under extreme conditions. Location and environment based services can be employed to detect and locate victims even when the core wireless communications networks are down. In this paper, a location awareness framework is proposed for victim signal detection and direction estimation. A wideband, multi-channel sensing method is proposed along with a two-antenna multiple signal classification (MUSIC) direction finding algorithm. The performance evaluation of proposed methods are discussed with both simulation and measurement results.

I. INTRODUCTION

Emergency call services such as enhanced 911 (E911) are designed to provide improved service including instant delivery of the victims location information to the local Public Safety Answering Point (PSAP) to which the caller is connected [1]. Taking the requirements of the 911 services into consideration, wireless service providers took incentive to employ precise location estimation techniques based on their network capacities and structures. Even though these systems are very helpful for limited number of calls and specific events, it is not possible maintain such services under extreme cases affecting an important section of the population living in an area.

During the disasters such as Pakistan and Australia vast area floods, California wild forest fires, and earthquakes in China, first hours and days are very crucial to save the lives of the victims. In this important period of time, the victims who are scattered around the disaster area would be looking for a way to communicate to get help. However, the wireless network infrastructure can be damaged during the extreme situation. Beside that, if they are calling to reach the 911 service, congestion due to the capacity limit of either network or PSAP can result latency or it may not be possible to provide victim's location information in a timely manner. On the other hand, location and environment awareness features of Cognitive radios (CRs) provide methods for wireless devices and networks enabling them to interact and learn from surrounding environment [2]. Public safety cognitive radios (PSCRs) can benefit from these features for victim signal detection and direction finding.

In this paper, a framework which consists of wideband sensing, signal identification, and direction finding modules is proposed for locating the victims and the first responders who can be lost or stuck in the operation area. The remainder of the paper is organized as follows: The proposed framework is detailed along with the system model in Section II. Performance metrics and the theoretical analysis of wideband sensing and direction finding methods are detailed in Section III. Section IV details the numerical results and Section V provides the conclusion.

II. SYSTEM MODEL

The public safety cognitive radios are deployed with the first responders operating on the field e.g., the disaster area and the first responders are distributed over the field. The PSCRs are assumed to be able to establish and maintain communications along the first responders in a robust manner and when there is no communications the radios are sensing the wireless spectrum periodically. Due to the power limitations, the sensing procedure can be organized in a sequential manner for the first responder groups in close proximity. Beside the multiple PSCRs attached to the PS officers, it is also assumed that there can be a single or multiple victims located in the disaster area who are trying to communicate or in communication for the rescue. Note that it is not possible to exactly know which channel is assigned to the call by the network (even the network is not operational, the final settings will be kept by the phone), thus it is assumed that PSCR is sensing multiple channels or for instance, if the network is a frequency-division duplexing (FDD) system, the whole uplink band can be sensed. The proposed system model is illustrated in Fig. 1.

If the sensing frequency band is $B_s = f_N - f_0$, where $f_0 < f_1 < \dots < f_N$, the time-domain transmit signal model can be defined as $x(t) = As(t)$. $s(t) = [s_1(t), \dots, s_M(t)]^T$ represents M independent signals where T denotes transposition. $s_m(t) = \sum_{k=-\infty}^{\infty} b_k u(t - kT_s) e^{j2\pi f_{c,m}t}$, where $m = 1, 2, \dots, M$, j is defined to be $\sqrt{-1}$, $u(t)$ is the pulse shaping filter for $f_m - f_{m-1}$ frequency range, b_k are the digitally modulated symbols, and $f_{c,m}$ is the center frequency given by $(f_{m-1} + f_m)/2$. A is $N \times M$ amplitude coefficient matrix with $a_{i,j}$ elements where $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$, $N \geq M$ and M is unknown to any of K , which denotes the number of PSCRs. In case

Fig. 1. System Model for Spectrum Sensing

no transmission is made by the j^{th} radio over i^{th} frequency band or if the PSCR can not receive any signal on that band $a_{i,j} = 0$.

After digital conversion and modulation, each independent signal passes through a different channel as indicated in the Fig. 1. The wireless channel can be modeled as a time-variant linear filter

$$h(t) = \sum_{i=1}^L h_i(t)\delta(t - \tau_i), \quad (1)$$

where L is the number of taps for the channel h and τ_i is delay for each tap. It is assumed that the taps are sample spaced and the channel is constant for a symbol but time-varying across multiple symbols. Note that one of the main differences between communications in disaster and non-disaster terrestrial is the channel propagation. Furthermore, channel propagation depends on the type of disaster environments (e.g. earthquake, flood, fire). This is an important and open issue that needs to be addressed. The transmitted signal is received along with noise at the receiver. Therefore, baseband model of the received signal at the input of the energy detector, after down conversion can be given as

$$y(t) = e^{j2\pi\xi t} [x(t) \star h(t)] + w(t) \quad (2)$$

$$= e^{j2\pi\xi t} \int x(\tau)h(t - \tau)d\tau + w(t), \quad (3)$$

where $\star$ is the convolution operation. When the channel model given in (1) is considered received signal becomes

$$y(t) = e^{j2\pi\xi t} \sum_{i=1}^L x(t - \tau_i)h_i(t) + w(t), \quad (4)$$

where ξ is frequency offset due to inaccurate frequency synchronization and $w(t)$ corresponds to additive white Gaussian noise (AWGN) sample with zero mean and variance of σ_w^2 . The received signal is sampled with sampling time of Δt at the analog to digital converter (ADC) and discrete-time received signal can be represented in vector notation by

$$y_n = [y(1), y(2), \dots, y(N)]. \quad (5)$$

Location awareness feature of the CR systems introduces digital signal processing techniques to estimate the emitter

Fig. 2. Modules of the Proposed Framework

locations [2]. Based on location awareness, a spectrum sensing and signal direction finding (DF) framework which will lead to the detection of the wireless signals along with the signal direction estimation is introduced. The proposed framework is illustrated in Fig. 2. After the wideband wireless spectrum is received by the PSCR, first, spectrum sensing method detects the active communications over the selected frequency bands. Second, the signal identification module (GSM, WCDMA, LTE, etc.) extracts the signal parameters such as bandwidth, center frequency, and signal to noise ratio (SNR). This module informs the signal direction estimation module along with the estimated parameters of the detected signals. The direction estimation module selects the optimum DF method based on the information received from the wideband spectrum sensing and signal identification modules and estimates the trajectory of the detected signals. Our work in this paper is limited to the wideband sensing and DF for further information on signal identification [3] can be consulted. Note that the proposed approach is also compatible to the spectrum surveying framework which was proposed in [4].

III. PERFORMANCE ANALYSIS

A. Wideband Sensing

Three signal processing methods used for spectrum sensing are matched filter, cyclostationary feature extraction, and energy detection [5]. First two are coherent detection techniques which provide better detection probability than non-coherent energy detection however, the coherent detectors require a priori information. Matched filter provides optimal detection by maximizing signal to noise ratio (SNR) but requires demodulation parameters. Cyclostationary feature detection can detect random signals depending on their cyclic features even if the signal is in the background of noise but it requires information about the cyclic characteristics [6]. Moreover, computational complexities of coherent methods are higher than the energy detection which leads to sensing duration and sensing period degradation [5]. Note that under emergency conditions the calls made will be quite shorter when compared the communications under normal conditions. Therefore, the sensing should be specifically quick and agile not to miss any communication attempt conducted by the victims.

Sensing over the whole uplink channel is necessary for the signal detection as indicated in Section II. Coherent detection techniques can not be applied to multiple channels at once. Therefore, beside the modulation parameters and cyclic features of the implemented technologies, the frequencies of the communications sub-channels that are assigned by the regulatory organizations over the operational area should be kept in a database and updated regularly. Under these conditions, applying matched filter and/or cyclostationary feature extraction to uplink sub-channels in a sequential way will lead

to loss of sensing time and reduce the victim signal detection probability. Moreover, implementation of hardware and the software that will execute the coherent detection function and the database will increase the cost and complexity of the public safety radio.

Energy detection technique is applied by setting a threshold for detecting the existence of the signal. Even though it's simpler than matched filter and cyclostationary feature detection, it requires at least $O(1/SNR^2)$ samples for detection [5] and it is susceptible to changing noise levels. It's not also possible to detect the direct sequence spread spectrum signals with very low SNR [7] however, the targeted signals for the PSCR are commercial signals that are not buried under noise intentionally¹. Therefore, despite the aforementioned drawbacks, energy detection can provide lower sensing period than other methods and it is applicability to multiple channels for wideband sensing makes it the best candidate for victim transmission sensing.

The decision problem for the energy detection scheme is formulated in a binary hypothesis and the thresholds satisfying certain detection and false alarm rates are derived in [8]. The theoretical analysis indicate that optimal threshold is not only dependent of the number of samples but also estimation of the noise variance and the SNR for each signal in the spectrum. Methods such as [9] proposed noise floor estimation via histogram and estimates of false alarm and detection thresholds are derived based on the noise variance of the side or empty bands [10]. However, especially under extreme conditions the characteristics of wireless channel changes [11] and noise floor can become non-stationary through the sensed spectrum. Beside that, in case of a wideband multiple signal scenario derived thresholds should be updated for each signal.

The problems related to the wideband sensing addressed and solutions based on wavelet approach [12], compressed sensing [13], and optimization of a bank of multiple narrowband detectors [14] are proposed. However even though the sensing performance improved, implementations of these methods over the PSCRs are difficult due to the complexity issues and the hardware/software requirements. On the other hand, in [8], we adopted the adaptive threshold method from the literature of image processing. The adaptive threshold method is employed to determine the level of image intensity boundary for converting the color or the gray scale images to the binary images. The process is called image binarization [15] and it defines the threshold as linear function of the first and second order statistics of the data. This approach finds its validation in the fact that the standard deviation indicates the dispersion level of the spectral data around the mean value.

The standard deviation coefficient has an important effect on the performance of the threshold as explored also in [16] and we investigated the range for the standard deviation coefficient taking the spectrum occupancy and SNR into consideration. Our research indicated that even though computation of the mean and the standard deviation at each point and adjusting the threshold accordingly will provide an advantage against non-stationary noise floor and changes in the signal due to the extreme conditions, because of the cumulatively increasing

nature of the threshold, it can not trace sharp changes in the frequency spectrum fast enough. Hence, the missed detection performance of the threshold will be deteriorated especially in the existence of multiple signals over the spectrum.

One way to improve the missed detection performance without losing the advantages of the adaptive threshold is to modify the threshold formulation introducing another coefficient to the first order term. Thus the two-parameter adaptive threshold can be written as

$$\lambda(i) = k\mu(i) + l\sigma(i) \quad \forall Y(i) \quad (6)$$

where

$$\mu(i) = \frac{1}{i} \sum_{p=1}^i Y(p), \quad (7)$$

$$\sigma(i) = \sqrt{\frac{1}{i} \sum_{p=1}^i (Y(p) - \mu(i))^2}, \quad (8)$$

$Y(i) = \sum_{n=1}^N y(n)e^{-j2\pi i \frac{n}{N}}$, and $i = 1, 2, \dots, N$. Modified two-parameter adaptive threshold eliminates the requirement of estimation of the noise variance and SNR for all possible signals in the spectrum improving the missed detection performance. In practice, the PSCR can plot a number of thresholds using a set of coefficients and the first responder can select the optimal threshold for once by visual inspection [17] and the coefficients of the selected threshold can be fixed for the following sensing processes. Note that, threshold will be different for each process but the mean and standard deviation coefficients will be the same.

B. Direction Finding

Various approaches and their applications are proposed to estimate the emitter location and for signal DF. One method is to measure the received signal strength indicators (RSSI) and mapping them to the measurement field for future predictions which is called fingerprinting [18]. Even though this method is simple, instead of a precise information location estimation based on signal strength gives a rough estimate for the emitter location. Beside that, under extreme conditions such as tornadoes and hurricanes the signal strength information becomes undependable. Time of arrival (TOA), angle of arrival (AOA), time difference of arrival (TDOA) based algorithms can provide precise location information however these algorithms need at least two receivers (or 2 transceiver hardware that can be embedded together), very low recording precision (to be able to hold the recording time of the signals with the precision of μ seconds), and coarse synchronization to be able to track the time of difference of arrival [19]. These requirements indicate a computationally expensive and precise process which is prone to the timing errors if the wireless signals and the receiver systems are not designed (such as global positioning system i.e, GPS) for solely location estimation purposes. On the other hand, interferometric DF methods can estimate the direction of the incoming signals by behaving as a pseudo-Doppler system switching between even number of antennas located at least half of the received signal wavelength, $\lambda/2$ far away from each other [20]. Even though interferometers are simpler than TOA/AOA based methods,

¹for instance, demodulation limit of GSM systems is 10 dB of SNR

for efficient DF at least four antennas are required and this makes them difficult to implement over PSCRs. However, multiple signal classification (MUSIC) algorithm provides asymptotically unbiased estimates directions of arrival (DOA) of the signals if at least two independent paths for the received signal can be provided [21]. This requirement can be satisfied by switching between two antennas which are $\lambda/2$ distanced while they are connected to the same receiver. Note that, two antennas satisfying these conditions can be implemented over the public safety radio or one of the antennas can be mounted over the gear of the public safety officers. When the MUSIC algorithm considered, (4) can be written in the matrix form as

$$Y(t) = DS(t) + W(t) \quad (9)$$

where

$$S(t) = [s_1(t), \dots, s_m(t)]^T, \quad (10)$$

$$D = [D_{\theta_1}, D_{\theta_2}, \dots, D_{\theta_m}], \quad (11)$$

and

$$D_{\theta_k} = [1, e^{-j\tau_k}, e^{-j2\tau_k}, \dots, e^{-j(K-1)\tau_k}]^T, \quad (12)$$

where T denotes the transpose, $\tau_k = 2\pi(d/\lambda)\sin\theta_k$, θ_k is the measured relative arrival angle, d is the antenna spacing, and $W(t) = [w_1(t), \dots, w_n(t)]^T$. Based on these preliminary formulations [22], the spatial covariance matrix is given by

$$R = E\{Y(t)Y'(t)\} = R_x + R_N \quad (13)$$

where $'$ is the complex conjugate transpose, $E\{\cdot\}$ is the expectation, I is the $K \times K$ identity matrix. In DF problems, estimating the DOA θ_k by eigenstructure methods are done by exploiting R , so that R_x has a range basis consisting of the direction vectors of source plane waves [22]. MUSIC method utilizes noise space eigenvectors of spatial covariance matrix corresponding to the smallest $K-M$ eigenvalues are orthogonal to all the source direction vectors defined by

$$D_{\theta_m}' u_i = 0, \quad i = p+1, p+2, \dots, L; \quad m = 1, \dots, p. \quad (14)$$

where D_{θ_m} are the path source direction vectors; and u_i is the i^{th} eigenvector which lies in the noise space. Considering these equations, source direction can be estimated as the peaks of the spatial spectrum defined by

$$P_{MUSIC}(\theta) = \frac{1}{D_{\theta}' (\sum_{i=p+1}^L u_i u_i') D_{\theta}} \quad (15)$$

IV. SIMULATION & MEASUREMENT RESULTS

A set of simulations are conducted by generating baseband signals which employ 16 QAM modulation with various symbol rates, each signal is consisting of 1024 symbols to investigate the performance of the proposed thresholding method. Combination of root raised cosine ($\alpha = 0.3$) and Gaussian filters are used for pulse shaping. The impacts of multipath channel is simulated in MATLAB by considering ITU-R M.1225 outdoor to indoor channel model which is compatible to the system model proposed in Section II. First, the detection performance of the adaptive threshold under different SNR levels is analyzed. The mean and standard deviation coefficients are selected to be optimal for the constant false alarm rate (CFAR) detector developed. The simulation results

Fig. 3. Probability of Detection vs. SNR

are illustrated in Fig. 3 for false alarm (P_F) rates of 0.01, 0.05, and 0.1.

On the other hand, mean and standard deviation coefficients, namely k and l are the main parameters which affects the detection performance of the threshold and the optimal coefficients are characterized by the distribution of the recorded spectrum. The range of their values can be investigated by taking the SNR and spectral occupancy ratio which is the ratio of the occupant signals to the whole sensed spectrum into consideration. Optimum values of k and l that satisfy $P_F = 0.01$ are illustrated (Fig.4 (a)) and (Fig.4 (b)) for various SNR levels. The simulation results indicate both parameters are slightly increasing with decreasing occupancy. Beside that, characteristically k is in general within the range of $[0, 1]$ and l is $[-10, 5]$ if the occupancy is between 0.5 and 0.2 independent of SNR.

Fig. 4. Optimum k and l values: SNR vs. Occupancy

A. Empirical Results

MUSIC is a theoretically well studied algorithm and for the performance analysis [21] can be consulted. In this section we will focus on the performance of the MUSIC method specifically for the two antenna single receiver system. We developed a measurement setup to test the performance of the wide-band sensing and DF algorithms. The measurement system is consists of two Anritsu 2000-1031 personal communications

service (PCS) band antennas [23], a Mini Circuits ZSWA-4-30DR switch [24], Parallax controller board with BS2-IC micro-controller [25] to control the switching mechanism, Anritsu MT8222A BTS Master spectrum analyzer with I/Q data recording capability, Dell Vostro 1520 laptop computer, power supplies, cables, and connectors. We generated GSM, IS-95, and WCDMA signals over the PCS uplink band (1850MHz - 1910MHz) and conduct the measurements. Note that, it is assumed that during the measurement process there is only a single victim radio transmission over the sensed spectrum. However based on the proposed framework in Section II, extension to the multiple signal scenario is straightforward. Fig. 5 depicts the angular direction estimation error of the two-

Fig. 5. MUSIC Direction Finding Estimation Error

antenna measurement system. If the SNR is low, the effect of the noise increase the prediction error however as the SNR increases error reduces below the 15 degrees. Note that the estimation error $E_{\theta_k} = E\{|\theta_k - \hat{\theta}_k|\}$, where $\hat{\theta}_k$ is the estimate of the actual signal direction. Note that optimum k and l values for GSM signal is in the range of [5, 6] due to low 0.0050 occupancy ratio over the PCS uplink, IS-95 has the occupancy ratio of 0.0313 and the coefficients range becomes [3, 4], for WCDMA 0.125 and [0, 1].

V. CONCLUSION

Safety and rescue operations under extreme cases can benefit from the prevalence of the wireless communications network to track the victims especially distributed over vast disaster areas. The victims may not be able to connect to the network due to congestion or network can be inoperable. However, the intelligent PSCR can detect and estimate the incoming direction of any emitter in the operation field. The proposed framework defines the spectrum sensing and DF requirements of the location aware PSCR. The simulation and measurement results indicate that two-antenna MUSIC algorithm along with the two-parameter adaptive threshold provides a simple and applicable solution for the victim location estimation problem. The performance of the proposed methods can also be improved by collaborative sensing and DF between the first responders over the field.

REFERENCES

- [1] "Federal Communications Commission, Wireless 911 Services". [Online]. Available: <http://www.fcc.gov/cgb/consumerfacts/wireless911srvc.html>

- [2] H. Celebi, H. Arslan, "Enabling Location and Environment Awareness in Cognitive Radios," *Elsevier Computer Communications-Special Issue on Advanced Location-Based Services*, 2008.
- [3] A. Gorcin, B. Thiagarajan, "A Signal Identification Application for Cognitive Radio," In *Proc. SDR Forum Technical Conference 2007*, Denver, CO, 5-7 November 2007.
- [4] D. Datla, A.M. Wyglinski, G.J. Minden, "A Spectrum Surveying Framework for Dynamic Spectrum Access Networks," In *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4158-4168, 2009.
- [5] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Spectrum Sensing Measurements of Pilot, Energy, and Collaborative Detection," In *Proc. IEEE Military Communications Conference* pp. 1-7, Washington D. C., U.S.A., 23-25 Oct. 2006.
- [6] A. Tonmukayakul, M. B. H. Weiss, "Secondary Use of Radio Spectrum: A feasibility Analysis," In *Proc. of TPRC Conference*, Quarter Hill Road, VA, Aug. 2004.
- [7] T. Yucek, H. Arslan, "A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications," In *IEEE Communications Surveys & Tutorials*, vol. 11, no. 1, pp. 116-130, 2009.
- [8] A. Gorcin, K.A. Qarage, H. Celebi, H. Arslan, "An Adaptive Threshold Method for Spectrum Sensing in Multi-channel Cognitive Radio Networks," In *Proc. IEEE 2010 17th International Conference on Telecommunications (ICT 2010)*, pp. 425-429, Doha, Qatar, 4-7 April 2010.
- [9] M. P. Oliveri, G. Barnett, A. Lackpour, A. Davis, and P. Ngo "A Scalable Dynamic Spectrum Allocation System with Interference Mitigation For Teams Of Spectrally Agile Software Defined Radios," In *New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2005)*, pp. 170-179, Baltimore, Maryland, U.S.A., Nov. 2005.
- [10] Z. Ye, G. Memik, J. Grosspietsch, "Energy Detection Using Estimated Noise Variance for Spectrum Sensing in Cognitive Radio Networks," In *IEEE Wireless Communications and Networking Conference, WCNC 2008* pp. 711-716, Las Vegas, Nevada, 31 March - 3 April, 2008.
- [11] N. Marchetti, Ed., "Telecommunications in Disaster Areas," ISBN 8792329489, River Publishers, 2010.
- [12] Z. Tian, G.B. Giannakis, "A Wavelet Approach to Wideband Spectrum Sensing for Cognitive Radios," In *IEEE 1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications, CrownCom 2006*, pp. 1-5, Mykonos Island, Greece, 8-10 June, 2006.
- [13] Z. Tian, G.B. Giannakis, "Compressed Sensing for Wideband Cognitive Radios," In *IEEE International Conference on Acoustics, Speech and Signal Processing, 2007. ICASSP 2007*, pp. 13571360, Honolulu, HI, U.S.A., 15-20 April, 2007.
- [14] Z. Quan, S. Cui, A. H. Sayed and H. V. Poor, "Optimal Multiband Joint Detection for Spectrum Sensing in Cognitive Radio Networks," In *IEEE Transactions on Communications* vol. 57, number 3, pp. 1128-1140, 2009.
- [15] M. Sezgin, B. Sankur, "A Survey over Image Thresholding Techniques and Quantitative Performance Evaluation," In *J. Electron. Imag.*, vol.13, no.1, pp. 146-165, Jan. 2004.
- [16] C. Ghosh, S. Pagadarai, D. Agrawal, A.M. Wyglinski, "A Framework for Statistical Wireless Spectrum Occupancy Modeling," In *IEEE Transactions on Wireless Communications*, vol. 9, no. 1, pp. 38-44, 2010.
- [17] M.A. McHenry, "NSF Spectrum Occupancy Measurements Project Summary," Shared Spectrum Co, Vienna, VA, Tech. Rep., Aug. 2005.
- [18] G. Sun, J. Chen, W. Guo, KJR Liu, "Signal processing techniques in network-aided positioning: a survey of state-of-the-art positioning designs," In *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 12-23, 2005.
- [19] W.C. Li, P. Wei, X.C. Xiao, "TDOA and T^2/R Radar Based Target Location Method and Performance Analysis," In *IEEE Radar, Sonar and Navigation Proceedings*, vol. 152, no. 3, pp. 219-223, 2005.
- [20] D. Peavey, and T. Ogunfunmi, "The Single Channel Interferometer Using a Pseudo-Doppler Direction Finding System", In *Proc. IEEE International Conference on Acoustics Speech and Signal Processing*, vol.5, pp. 4129-4132, Munich, Germany, 21 - 24 Apr. 1997.
- [21] R. Schmidt, "Multiple Emitter Location and Signal Parameter Estimation," In *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, 1986.
- [22] Kung, S. and Lo, C. and Foka, R., "A Toeplitz Approximation Approach to Coherent Source Direction Finding", In *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP'86)*, vol. 11, pp. 193 - 196, Tokyo, Japan, Apr. 1986.
- [23] Web: [<http://www.anritsu.com/en-gb/Products-Solutions/Accessories/Antennas-Directional-GPS-and-Portable.aspx>]
- [24] Web: [<http://www.minicircuits.com/pdfs/ZSWA-4-30DR.pdf>]
- [25] Web: [<http://www.parallax.com/StoreSearchResults/tabid/768/txtSearch/List/0/SortField/4/ProductID/297/Default.aspx>]