Adaptive Rate Transmission for Spectrum Sharing System with Quantized Channel State Information

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***Abstract*—The capacity of a secondary link in spectrum sharing systems has been recently investigated in fading environments. In particular, the secondary transmitter is allowed to adapt its power and rate to maximize its capacity subject to the constraint of maximum interference level allowed at the primary receiver. In most of the literature, it was assumed that estimates of the channel state information (CSI) of the secondary link and the interference level are made available at the secondary transmitter via an infinite-resolution feedback links between the secondary/primary receivers and the secondary transmitter. However, the assumption of having infinite resolution feedback links is not always practical as it requires an excessive amount of bandwidth. In this paper we develop a framework for optimizing the performance of the secondary link in terms of the average spectral efficiency assuming quantized CSI available at the secondary transmitter. We develop a computationally efficient algorithm for optimally quantizing the CSI and finding the optimal power and rate employed at the cognitive transmitter for each quantized CSI level so as to maximize the average spectral efficiency. Our results give the number of bits required to represent the CSI sufficient to achieve almost the maximum average spectral efficiency attained using full knowledge of the CSI for Rayleigh fading channels.**

I. Introduction

Spectrum-sharing system has been recently introduced as an efficient means for utilizing the scarce spectrum by allowing the secondary users to share the spectrum with licensed primary users. In particular, the secondary user is allowed to use the spectrum of the primary link under the constraint of not increasing the average interference level at the primary receiver above a predetermined value. Adaptive modulation can be utilized as a power-efficient technique for maximizing the capacity of the secondary link while satisfying the interference constraint by allowing the secondary user to adapt its power and rate according to the channel state information (CSI) of the secondary channel between the secondary transmitter and the secondary receiver, and the interference channel between the secondary transmitter and the primary receiver.

The capacity of the secondary link in spectrum sharing systems has been first studied in [1] under the constraint of average interference level. Then expressions for the capacity results have been developed under various sets of constraints such as peak/average transmit power and peak/average interference level in [2]. In both papers, it was assumed that the secondary transmitter has full knowledge of the CSI of the secondary and interference links. Assuming imperfect CSI available at the secondary transmitter, the capacity of the secondary link under the constraint of peak and average interference outage level has been developed [3], [4]. It was assumed that the imperfect CSI is represented by an infinite resolution estimate of the CSI in addition to an estimation error. Such assumption requires the need for an infinite resolution feedback link which is not always practical as it requires an excessive amount of bandwidth. In [4], the effect of quantizing the CSI of the interference channel on the capacity of the secondary link was studied, however, a mid-rise uniform quantizer was assumed which is not necessarily the optimal quantizer that maximizes the capacity.

In this paper, we consider the problem of maximizing the average spectral efficiency of a secondary link in fading environment assuming quantized CSI of the secondary and interference links. Under the assumption of quantized CSI made available at the secondary transmitter, our objective is to find the optimal CSI quantizers employed at the primary and the secondary receivers and the associated discrete power and rate level associated with each quantized CSI level so as to maximize the performance of the secondary link in terms of the average spectral efficiency. This problem has been well investigated for the case of single slowly fading wireless channel under the assumption of average transmit power. It was shown that the optimal design of the CSI quantizer and the associated power and rates results in achieving average spectral efficiency values almost close to the Shanon capacity [5], [6], [7].

We note that the problem of interest has been first addressed in [8] where a framework was developed for finding optimal power and rates assuming a fixed set of CSI quantizers of the secondary and interference links. Then numerical search techniques were applied for finding the optimal quantizer thresholds that are not computationally efficient for high number of quantizer bits. In addition, the CSI quantizer for the secondary link were assumed to be fixed and the search focused on optimal quantizer for the interference link and the associated discrete power and rates levels.

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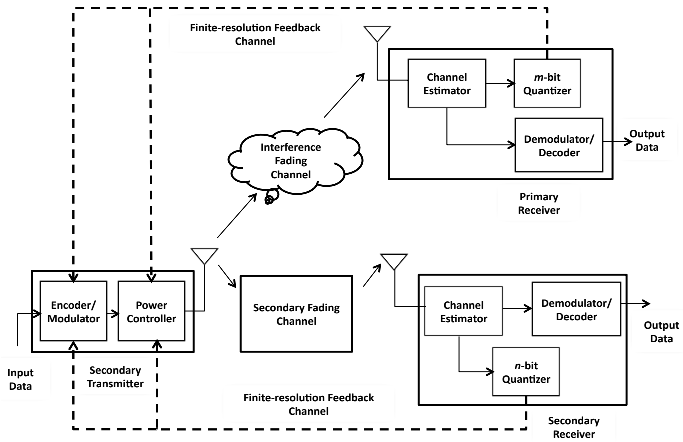

Figure 1. System block diagram for the spectrum sharing system based on quantized channel state information .

However, in this paper, we develop a framework for finding the optimal CSI quantizers of the both the secondary and interference links. We also develop a computationally efficient algorithm for jointly finding the optimal CSI quantizers of the secondary and interference links and the associated discrete power and rate values. We also find the number of bits sufficient to achieve the average spectral efficiency almost attained using full knowledge of CSI of the secondary and interference links.

The remainder of this paper is organized as follows. In Sec. II and Sec. III, we present the system model as well as the problem formulation. In Sec. IV, we present iterative-based techniques for finding the optimal rates and the optimal CSI quantizers of both the secondary and interference links. In Sec. V, we present numerical results to assess the performance of the developed techniques and determine the number of bits required to achieve average spectral efficiency values attained using full knowledge of the CSI for Rayleigh fading channels. Finally, we conclude the paper in Sec. VI.

II. System Model

In this paper, we consider a network setting depicted in Fig. II whereby a secondary transmitter is communicating with its secondary destination over a secondary channel. The secondary transmitter is allowed to share the spectrum used by a primary network. Such simultaneous transmission results in an interference power level observed at the primary receiver due to the existence of a channel between the secondary transmitter and the primary receiver; namely, the interference channel. We assume discrete-time Rayleigh fading channels where the signal-to-noise ratio (SNR) of the secondary and interference link are given by γ_s and γ_p, respectively, with mean values given by $\bar{\gamma_s}$ and $\bar{\gamma_p}$, respectively. We assume that the CSI of the secondary and interference channels are estimated at the secondary and primary receivers, respectively. The CSI of the secondary and interference channels are then quantized using n-bit quantizer and m-bit quantizer, respectively, then fed back to the secondary transmitter over error-free finite-resolution feedback channels. We note that our assumption of relying only on the quantized CSI of the interference link stems from the fact that the primary base station will be required to estimate the channels of all secondary nodes involved in communication in addition to its primary transmitter. Therefore, in order to limit the amount of feedback bandwidth needed for conveying the CSI of the interference channels, it is desirable to rely on quantized CSI.

We note that the assumption of the primary receiver aids in estimating the interference channel has been considered in almost all of the literature related to spectrum-sharing systems [1], [2], [3], [9] as it is a crucial assumption in order to satisfy the interference power level constraint at the primary receiver. An example of a method for obtaining these estimates by the primary receiver is shown in [9] whereby the secondary transmitter is allowed to transmit a pilot signal every frame to the primary receiver. This pilot signal can be used by the primary receiver to estimate the CSI of the interference channel.

A. *Quantizer Model*

We consider quantizing γ_s of the secondary channel and γ_p of the interference channel using n-bit quantizer with $N = 2^n$ quantizer thresholds denoted by $\{\gamma_{s,i}\}$ and m-bit quantizer with $M = 2^m$ quantizer thresholds denoted by $\{\gamma_{p,j}\}$, respectively. As a result, the quantizer regions for the secondary channel are given by

$$I_i = [\gamma_{s,i}, \gamma_{s,i+1}) \tag{1}$$

with quantizer thresholds satisfying $\gamma_{s,0} = 0 < \gamma_{s,1} < \cdots < \gamma_{s,N+1} = \infty$. While the quantizer regions for the interference channel are given by

$$I_j = [\gamma_{p,j}, \gamma_{p,j+1}) \tag{2}$$

with quantizer thresholds satisfying $\gamma_{p,0} = 0 < \gamma_{p,1} < \cdots < \gamma_{p,M+1} = \infty$. We adopt the scenario of zero information outage where the secondary user is not allowed to transmit below $\gamma_{s,1}$ given any value of γ_p [7]. In addition we assume, without loss of generality, no transmission is allowed above $\gamma_{p,M}$.

By combining the quantizer regions defined in equations (1) and (2), we define the following two dimensional quantizer regions

$$I_{i,j} = ([\gamma_{s,i}, \gamma_{s,i+1}), [\gamma_{p,j}, \gamma_{p,j+1})), \tag{3}$$

that represents the quantized CSI of the secondary and interference channels. Our objective in this paper is to find the optimal discrete rate and power employed at each region $I_{i,j}$ and the associated optimal quantizer thresholds so as to maximize the average spectral efficiency subject to interference power constraint at the primary receiver.

III. Problem Formulation

In this paper, we adopt the average spectral efficiency as a performance metric for our problem. We assume that the secondary employs discrete power $p_{i,j}$ and discrete rates $R_{i,j}$

at each interval $I_{i,j}$. Assuming capacity-achieving code the discrete rate $R_{i,j}$ is given by

$$R_{i,j} = \log_2\left(1 + p_{i,j}\gamma_{s,i}\right). \quad (4)$$

By averaging the discrete rates over all possible quantizer regions, the average spectral efficiency can be given as follows

$$\bar{\eta} = \sum_{i=1}^{N}\sum_{j=0}^{M-1} R_{i,j}[F_s(\gamma_{s,i+1}) - F_s(\gamma_{s,i})] \quad (5)$$

where $F_s(\cdot)$ and $F_p(\cdot)$ are the cumulative distribution functions (CDF) of the secondary and interference SNRs, respectively.

We consider the problem of maximizing the average spectral efficiency subject to average interference power allowed at the primary receiver, *i.e.*

$$\max_{\{\gamma_{s,i},\gamma_{p,j},p_{i,j}\}} \bar{\eta}$$

subject to the constraints

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=0}^{M-1}[p_{i,j}[F_s(\gamma_{s,i+1}) - F_s(\gamma_{s,i})] \\ &\qquad [H_p(\gamma_{p,j+1}) - H_p(\gamma_{p,j})]] \leq Q, \\ &\qquad p_{i,j} \geq 0, \end{aligned} \quad (6)$$

where $H_p(\gamma_{p,j}) = \int_0^{\gamma_{p,j}} \gamma_p f_p(\gamma_p) d\gamma_p$, and Q is the maximum average interference power allowed at the primary receiver.

The above problem is a multidimensional optimization problem that involves finding the optimal thresholds as well as the discrete rates that maximize the average spectral efficiency. Unfortunately, we can show that the above problem is not convex and hence can not be solved using standard convex optimization technique. In the next section, we develop an iterative-based algorithm that exhibits fast convergence which jointly find the discrete power levels as well as the optimal quantizer thresholds that maximize the average spectral efficiency and satisfy the interference constraint.

IV. Iterative-based System Design Algorithm

In this section, we present an efficient and fast algorithm to find the optimal discrete power values $p_{i,j}$ and the quantizer thresholds. By introducing the Lagrangian multiplier λ associated with the interference power constraint, we focus on solving the dual problem

$$\begin{aligned} &\min_{\lambda}\max_{\{\gamma_{s,i},\gamma_{p,j},p_{i,j}\}} \sum_{i=1}^{N}\sum_{j=0}^{M-1} \log_2(1+p_{i,j}\gamma_{s,i})[F_s(\gamma_{s,i+1}) \\ &\quad -F_s(\gamma_{s,i})][F_p(\gamma_{p,j+1}) - F_p(\gamma_{p,j})] - \lambda\sum_{i=1}^{N}\sum_{j=0}^{M-1}[p_{i,j} \\ &\quad [F_s(\gamma_{s,i+1}) - F_s(\gamma_{s,i})]\,[H_p(\gamma_{p,j+1}) - H_p(\gamma_{p,j})]]. \end{aligned} \quad (7)$$

In solving the problem in (7), we first find the discrete values of $p_{i,j}$ assuming a fixed set of quantizer thresholds $\{\gamma_{s,i},\gamma_{p,j}\}$. By differentiating (7) with respect to $p_{i,j}$ and setting the derivative to zero the value of optimal $p_{i,j}$ is given by the following water-filling algorithm

$$p_{i,j} = \left[\frac{\log_2(e)[F_p(\gamma_{p,j+1}) - F_p(\gamma_{p,j})]}{\lambda[H_p(\gamma_{p,j+1}) - H_p(\gamma_{p,j})]} - \frac{1}{\gamma_{s,i}}\right]^+ \quad (8)$$

where the value of λ can be obtained by applying the interference constraint and $[\cdot]^+ = \max(\cdot, 0)$. In the next step, we fix the values of $p_{i,j}$ and λ and the quantizer thresholds for the secondary channel $\{\gamma_{s,i}\}$, then find the values of the quantizer thresholds for the interference channel $\{\gamma_{p,j}\}$ that maximize (7). In particular, by setting the first derivative of (7) with respect to $\gamma_{p,j}$, we obtain the following equation

$$\gamma_{p,j} = \frac{\sum_{i\in\mathcal{I}} \log_2\left(\frac{1+p_{i,j-1}\gamma_{s,i}}{1+p_{i,j}\gamma_{s,i}}\right)(F_s(\gamma_{s,i+1}) - F_s(\gamma_{s,i}))}{\lambda\sum_{i\in\mathcal{I}}(p_{i,j-1} - p_{i,j})(F_s(\gamma_{s,i+1}) - F_s(\gamma_{s,i}))}, \quad (9)$$

where $\mathcal{I}$ is the set of the index i for which $p_{i,j}$ are strictly positive. By careful investigation of Eq. (9), we note that, at each iteration, the value of each quantizer threshold for the interference channel does not depend on the values of the other interference thresholds. Instead it depends only on the values of $p_{i,j}$ and the quantizer thresholds for the secondary channel$\{\gamma_{s,i}\}$. This results in reducing the complexity of finding the values of the interference thresholds.

In the final step, similarly by fixing the values of $p_{i,j}$ and λ and the quantizer thresholds for the interference channel $\{\gamma_{p,j}\}$, we can find the values of the quantizer thresholds for the secondary channels that maximize (7) as follows

$$\begin{aligned} F_s(\gamma_{s,i+1}) &= F_s(\gamma_{s,i}) + \frac{f_s(\gamma_{s,i})}{\log_2(e)\sum_{j=0}^{M-1}\frac{p_{i,j}[F_p(\gamma_{p,j+1})-F_p(\gamma_{p,j})]}{1+p_{i,j}\gamma_{s,i}}} \\ &\Bigg[\sum_{j=0}^{M-1}\log_2\Big(\frac{1+p_{i,j}\gamma_{s,i}}{1+p_{i-1,j}\gamma_{s,i-1}}\Big)[F_p(\gamma_{p,j+1}) - F_p(\gamma_{p,j})] \\ &+ \lambda\sum_{j=0}^{M-1}[p_{i,j} - p_{i-1,j}][H_p(\gamma_{p,j+1}) - H_p(\gamma_{p,j})]\Bigg] \end{aligned} \quad (10)$$

where $f_s(.)$ denotes the probability density function of the secondary SNR. The above equation can be used to find the quantizer thresholds for the secondary thresholds by applying a simple numerical iterative technique by exploiting the fact that $\gamma_{s,0} = 0$ and $\gamma_{s,N+1} = \infty$. Specifically, we can first an initial value for $\gamma_{s,1}$ then using (10) to find the rest of the quantizer thresholds $\gamma_{s,i}$ iteratively. Since $\gamma_{s,N+1} = \infty$, we check whether $F_s(\gamma_{s,N+1})$ is equal to one. If not, we can either increase or decrease the initial value of $\gamma_{s,1}$ if $F_s(\gamma_{s,N+1})$ is less or greater than one, respectively. We repeat these steps until we find the quantizer thresholds.

After finding the values of $p_{i,j}$, $\gamma_{s,i}$ and $\gamma_{p,j}$ using the above three steps, these steps are repeated until these values converge. These steps for solving (7) are described by the following iterative algorithm

Our numerical results show that the algorithm converges to the same optimal values of $p_{i,j}$, $\gamma_{p,j}$ and $\gamma_{s,i}$ independent of the initial values. We finally note that the above algorithm does

Algorithm 1 Iterative algorithm for finding the optimal $p_{i,j}$, $\gamma_{s,i}$ and $\gamma_{p,j}$

Initialize $k = 0$, fix arbitrary $\gamma_{p,j}^0$ and $\gamma_{s,i}^0$,
solve for $p_{i,j}^0$ and λ^0 using (8) by the water-filling algorithm.
repeat
  Fix $p_{i,j}^k$, λ^k and $\gamma_{s,i}^{k-1}$,
  solve for $\gamma_{p,j}^k$ using (9);
  Fix $p_{i,j}^k$, λ^k and $\gamma_{p,j}^{k-1}$,
  solve for $\gamma_{s,i}^k$ using (10)
  increment k
until $p_{i,j}$, $\gamma_{p,j}$ and $\gamma_{s,i}$ converge.

not guarantee that the obtained rates and thresholds are globally optimal. However, based on comparing our results with those achieved using an exhaustive search shown in [8], we can argue that the iterative algorithm achieves the maximum average spectral efficiency attained using an exhaustive search method.

V. Numerical Results

In this section, we present numerical results that assess the performance of the proposed iterative algorithm. We also determine the optimal number of quantizer bits required to represent the secondary and interference channels so as to achieve the maximum average spectral efficiency attained using full knowledge of the CSI.

Fig. 2 and Fig. 3 illustrate the convergence behavior of the secondary and interference thresholds, respectively, attained using the iterative algorithm for the values of $N = 4$ $(n = 2)$ bits, $M = 4$ $(m = 2)$ bits, $Q = 0$ dB and $\bar{\gamma}_p = 1$ dB and $\bar{\gamma}_s = 10$ dB. It is evident that the algorithm shows fast convergence as it requires about 30 iterations compared to the exhaustive search method shown in [8]. We note that the algorithm shows even faster convergence for lower values of Q.

Fig. 4 depicts the maximum average spectral efficiency as a function of M (the number of quantizer levels for the interference channel) for different values of N (the number of quantizer levels for the secondary channel) at $\bar{\gamma}_p = 1$ dB and $\bar{\gamma}_s = 10$ dB. The figure shows two sets of curves; the upper and lower set of curves represent the case for $Q = 10$ dB (weak interference constraint) and $Q = 0$ dB (strong interference constraint), respectively. For the sake of comparison, the average spectral efficiency is compared to the capacity of the system assuming full knowledge of CSI available at the secondary transmitter developed in [1]. Fig. V shows that the $N = 16$ (4-bit quantizers) and $M = 8$ (3-bit quantizers) suffice to achieve an average spectral efficiency almost close to the capacity values developed in [1] assuming continuous rate adaptation for Rayleigh fading channels. The figure also shows that such selection for the number of bits is not affected by the value of Q. We also note that for strong interference constraint ($Q = 0$ dB) which is considered a practical scenario, selecting $M = 4$ (2-bit quantizer) for the interference channel almost attain the same average spectral efficiency compared to $M = 8$.

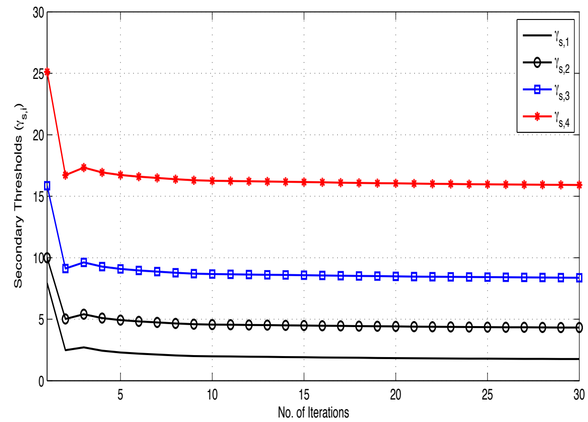

Figure 2. Convergence behavior of the quantizer thresholds $\{\gamma_{s,i}\}$ for the secondary channel obtained using algorithm (1) for $N = 4(n = 2)$ bits, $M = 4(m = 2)$ bits, $Q = 0$dB and $\bar{\gamma}_p = 1$dB and $\bar{\gamma}_s = 10$dB..

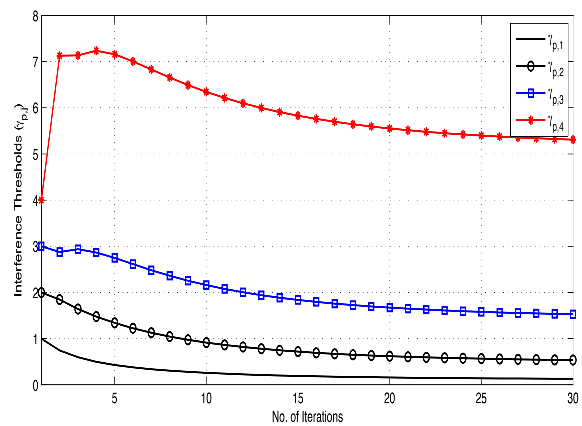

Figure 3. Convergence behavior of the quantizer thresholds $\{\gamma_{p,j}\}$ for the interference channel obtained using algorithm (1) for $N = 4$ $(n = 2)$ bits, $M = 4$ $(m = 2)$ bits, $Q = 0$ dB and $\bar{\gamma}_p = 1$ dB and $\bar{\gamma}_s = 10$ dB.

Fig. 5 depicts the average spectral efficiency as a function of the average SNR of the interference channel $\bar{\gamma}_p$ for different values of the average SNR of the secondary channel $\bar{\gamma}_s = 5, 10, 15$ dB, and $N = 16$, $M = 8$ and $Q = 0$ dB. The figures compare the spectral efficiency against the capacity of continuous rate spectrum sharing system. As our results reveal, the selection of $N = 16$ and $M = 8$ is still sufficient to represent the CSI of the secondary and interference channels, respectively for different values of $\bar{\gamma}_s$ and $\bar{\gamma}_p$. The figure also shows that the average spectral efficiency decreases almost linearly with $\bar{\gamma}_p$ in dB.

Fig. 6 depicts the probability of no transmission (outage probability) defined as

$$P_{\text{out}} = 1 - (1 - \Pr(\gamma_s < \gamma_{s,1}))(1 - \Pr(\gamma_p > \gamma_{p,M})) \quad (11)$$

as a function of $\bar{\gamma_p}$ for different values of $\bar{\gamma_s} = 5, 10, 15$ dB, and $N = 16$, $M = 8$ and $Q = 0$ dB. As the figure reveals, the outage probability increases as $\bar{\gamma_p}$ increases for a fixed value of $\bar{\gamma_s}$. In addition as the values of $\bar{\gamma_s}$ decreases, the outage probability suffers from higher increase as $\bar{\gamma_p}$ increases.

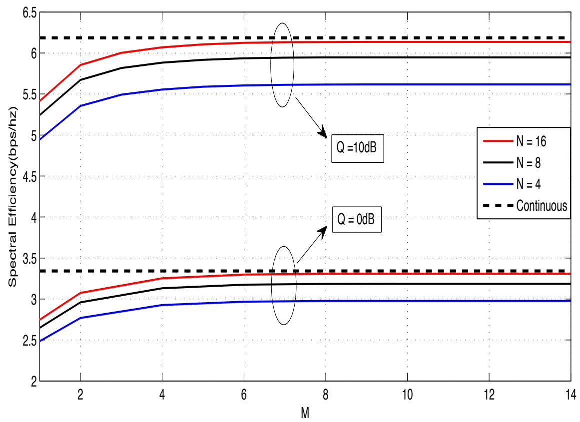

Figure 4. Average spectral efficiency as a function of M for different values of N = 4,8,16 at $\bar{\gamma}_p = 1$ dB and $\bar{\gamma}_s = 10$ dB. The upper and lower set of curves represent the case for $Q = 10$ dB (weak interference constraint) and $Q = 0$ dB (strong interference constraint), respectively.

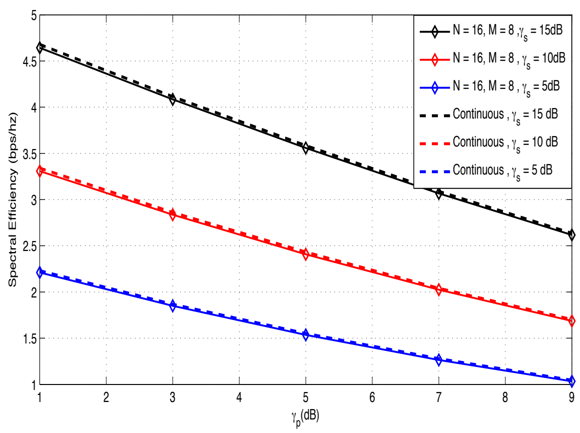

Figure 5. Average spectral efficiency as a function of the average SNR of the interference channel $\bar{\gamma_p}$ for different values of the average SNR of the secondary channel $\bar{\gamma_s} = 5, 10, 15$ dB and $N = 16$, $M = 8$ and $Q = 0$ dB.

VI. Conclusions

In this paper, we consider the problem of maximizing the average spectral efficiency of a secondary link employing discrete power and rate in spectrum-sharing systems assuming quantized CSI of the secondary and interference channels. We presented an iterative algorithm for finding the optimal quantizers of the secondary and interference CSI and the discrete power and rate employed within each quantizer levels. We have shown that a *four bit* level representation of the secondary CSI and a *three bit* level representation of the interference CSI attains values of average spectral efficiency almost close to the capacity values of spectrum-sharing systems.

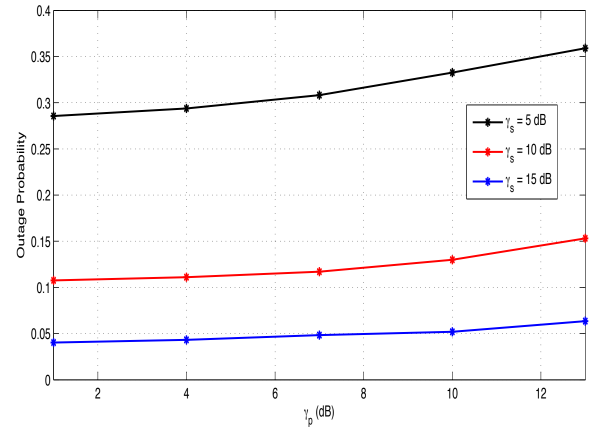

Figure 6. Outage probability versus $\bar{\gamma_p}$ for different values of $\bar{\gamma_s} = 5, 10, 15$ dB, $N = 16$, $M = 8$ and $Q = 0$ dB.

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