General Order Antenna Selection in MIMO Cooperative Relay Network[‡]

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Abstract—In this paper, we apply General Order Statistics (GOS) theorem to multi-antenna (MIMO) dual-hop amplify-forward fixed-gain cooperative relay network. General order link, unlike the best one, is selected for the transmission/receiving at each hop. The closed-form expressions of SNR (Signal-to-Noise-Ratio) statistics such as CDF (Cumulative Distribution Function), PDF (Probability Density Function), MGF (Moment Generating Function) and Generalized Moments are derived. The results are used to investigate two scenarios - presence of 1) the direct link, and 2) the multiple relays. The analysis is confirmed with computer simulation, and facilitates to quantify the performance loss when lower-order antenna is selected at any node.

I. INTRODUCTION

The cooperative relay networks with multi-antenna terminals has attracted growing research attention as shown by recent papers [1]- [7]. Considering a dual-hop amplify-forward relay scenario, these works presented end-end system performance with assumption of multi-antenna nodes at source/destination or relay. MRC (Maximal Ratio Combining) and TB (Transmit Beamforming) was used [1] to exploit spatial diversity at multi-antenna relay. A single-antenna relay and multi-antenna end-nodes with TB was analyzed in [2]; was extended to TAS/MRC in [4]; and to fixed-gain relay in [3]. MRC/SC (Selection Combining) and TAS were employed at multi-antenna relay in [5]. Bit Error Rate (BER) [6], and Outage and Symbol Error Rate (SER) [7] were evaluated in MIMO dual-hop relay with antenna selection. The antenna selection, which offers a cost-effective alternative to beamforming, was also recently used in multi-hop [9] MIMO relay networks. The idea of antenna selection comes from well-known theory of Ordered Statistics. The fundamental General Order Statistics (GOS) has been applied recently for system analysis in [8]- [11] where not only the highest but in general n^{th} order statistics are of interest. As [10] noted that n^{th} statistics is often required in signal detection/estimation. Another scenario where n^{th} statistics may be useful is in the evaluation of performance loss when the receiver/transmitter make error in selecting the best antenna. [11].

We extend [6] [7] analysis to include GOS in MIMO dual-hop amplify-forward system, where n_1^{th} and n_2^{th} order link (antenna-pair) are chosen at transmitter and receiver

respectively. All the channels and the links are subject to independently and identically distributed (i.i.d.) Rayleigh fading. We make following contributions to the current state-of-the-art knowledge:

- 1) Unlike in [6] [7], we assume a fixed-gain amplification at the relay. The fixed-gain relaying offers less-complicated alternative to the variable-gain scheme [12].
- 2) General Order SNR statistics such as CDF (cumulative distribution function (CDF), PDF (Probability Density Function), MGF (Moment Generating Function), and General Moments
- 3) Apply SNR statistics to investigate the presence of the direct link, and the multiple relays

From these results, special case e.g. conventional antenna selection, where the best antenna is chosen at both ends of the link, can be obtained. The analysis is validated through Monte-Carlo simulation.

The rest of the paper is organized as follows: Next section briefly describes the system model under consideration. In the following section, the theorem of GOS is invoked for Rayleigh distributed fading, and applied to derive General Order SNR statistics. Section IV extends the analysis to take account of the direct link and the multiple relays. Thereafter, some key findings are illustrated with numerical plots. Finally, the last section summarizes the main contribution and ends the paper.

Fig. 1: A MIMO Dual-Hop Relay System

[‡]This work is supported by Qatar National Research Fund (QNRF) grant through National Priority Research Program (NPRP) No. 08-577-2-241. QNRF is an initiative of Qatar Foundation.

II. SYSTEM DESCRIPTION

A dual-hop relay system, as shown in Fig. 1, consists of the source S sending signals towards the destination D via the assistance of the relay R . In Amplify-Forward (AF) relay systems, R amplifies the received signal before forwarding it to the D node. The total transmission involves two time-slots. The source has M_s antennas where the relay and the destination are equipped with M_r , and M_d antennas. Note that unlike in [6], we assumed same set of antennas at Relay for receiving and transmission. $\mathbf{H}_1$ denotes $M_r \times M_s$ channel matrix for the first hop whereas $\mathbf{H}_2$ is $M_d \times M_r$ matrix for the second hop, both matrix elements are i.i.d complex Gaussian random variables (CGRVs) with mean zero and variance 0.5 per dimension. The channel elements are ordered in the decreasing order of their absolute magnitudes. The n_1^{th} and n_2^{th} best links (Tx/Rx antenna pairs) are then selected for transmission in $S \rightarrow R$ and $R \rightarrow D$ hops respectively, therefore corresponds to the channel gains h_{n_1} and h_{n_2} .

The received signal at R is,

$$y_r = h_{n_1}x + n_r \quad (1)$$

where x is the transmit signal with normalized power, n_r is the additive white Gaussian noise vector with power N_0 at the relay antenna. The R amplifies the signal by G and transmits it through the n_2^{th} best link to the D . Therefore we can write the signal at D as,

$$\begin{aligned} y_d &= h_{n_2}Gy_r + n_d \\ &= Gh_{n_2}h_{n_1}x + Gh_{n_2}n_r + n_d \end{aligned} \quad (2)$$

n_d is the AWGN noise at the destination with same power N_0 . For a fixed-gain relaying, the equivalent SNR can be shown as [12, eqn. 6],

$$\gamma_{eq} = \frac{\gamma_1\gamma_2}{C + \gamma_2} \quad (3)$$

where the fixed Gain $G^2 = 1/(CN_0)$, C being a constant; the first hop SNR $\gamma_1 = \frac{1}{N_0}|h_{n_1}|^2 = \bar{\gamma}_1|h_{n_1}|^2$, and the second hop SNR $\gamma_2 = \frac{1}{N_0}|h_{n_2}|^2 = \bar{\gamma}_2|h_{n_2}|^2$.

III. SNR STATISTICS ANALYSIS

In this section, we study the statistical behaviour of the general order antenna selection (GOAS) multi-antenna dual-hop amplify-forward fixed-gain cooperative relay network. In particular, we derive closed form expressions for cumulative distribution function, probability distribution function, moment generation function, and general moments for the relayed link.

A. Statistical characterization of the received SNR

In this subsection, we derive closed form expressions for CDF, and PDF of γ_{eq} defined in (3). From the CDF expression, we easily derive the outage probability. The outage probability is an important system performance metric, and defined as the probability that the instantaneous SNR falls below a predefined threshold, γ_{th} . The CDF of γ_{th} is given by the following theorem.

Fig. 2: Cumulative Distribution Function for General Order Antenna Selection in MIMO Relay, '•' indicates Monte-Carlo simulation points

Theorem 1: The c.d.f. of γ_{eq} is given by

$$\begin{aligned} F_{\gamma_{eq}}(\gamma) &= 1 - 2M_{sr}M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1} \\ &\sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} \\ &\frac{(-1)^{k_1+k_2}}{e^{(n_1+k_1)\frac{\gamma}{\bar{\gamma}_1}}} \sqrt{\frac{C\gamma}{(n_1+k_1)(n_2+k_2)\bar{\gamma}_1\bar{\gamma}_2}} K_1(\lambda) \end{aligned} \quad (4)$$

where $K_1(\cdot)$ denotes the modified Bessel of second kind, $\lambda = 2\sqrt{\frac{(n_1+k_1)(n_2+k_2)C\gamma}{\bar{\gamma}_1\bar{\gamma}_2}}$, $M_{sr} = M_sM_r$, and $M_{rd} = M_rM_d$.

Proof: See Appendix I. ■

Corollary 1: The PDF of γ_{eq} can be obtained by taking derivative of (22) with respect to γ as follows

$$\begin{aligned} f_{\gamma_{eq}}^{GOS}(\gamma) &= 2M_{sr}M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1} \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \\ &\binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} \sqrt{\frac{C\gamma}{(n_1+k_1)(n_2+k_2)\bar{\gamma}_1\bar{\gamma}_2}} \\ &(-1)^{k_1+k_2} \exp\left(-\frac{\gamma(n_1+k_1)}{\bar{\gamma}_1}\right) \left[\frac{n_1+k_1}{\bar{\gamma}_1} K_1(\lambda) + \frac{\lambda}{2\gamma} K_0(\lambda) \right] \end{aligned} \quad (5)$$

Proof: The proof is straightforward, by applying the identity $z \frac{d}{dz} K_v(z) + vK_v(z) + zK_{v-1}(z) = 0$ [15, 8.486-12]. ■

Corollary 2: The outage probability of γ_{eq} for general order antenna selection multi-antenna dual-hop amplify-forward fixed-gain cooperative relay network can be given by substituting $\gamma = \gamma_{th}$.

For special cases, when $M_s = M_r = M_d = 1$ indicating single-antenna nodes, (4) specializes to [12, eqn. 9]. Furthermore when $n_1 = n_2 = 1$ (highest order statistics i.e. best

Fig. 3: Probability Density Function for General Order Antenna Selection in MIMO Relay for same total number of antennas

Tx/Rx antennas pair in both hops), we obtain the CDF for antenna selection in MIMO dual-hop amplify-forward system with a fixed-gain relay. Note that [6] [7] obtained results for an ideal-gain relay.

Fig. 2 shows the CDF plot for varying order statistics. The first and the second hop average SNRs are assumed equal, i.e. $\bar{\gamma}_1 = \bar{\gamma}_2 = 5\text{dB}$, and the number of antennas at relay $M = 4$. One can see that how the lower order statistics result a loss in the system performance. The Monte-Carlo simulation results validate the analysis. Fig. 3 shows the PDFs for two different settings with a total number of antenna in the system fixed to 6 - first setting involves evenly distributed antenna i.e. $M_s = M_r = M_d = 2$, and; in the second one there are 4 antennas at the relay and 1 antenna each at the source and destination, i.e. $M_s = M_d = 1, M_r = 4$. The system performance is similar in both cases as illustrated in Fig. 3.

B. Moment generating function (MGF)

MGF is useful to compute error rates as shown in [16]. Since MGF $M_{\gamma_{eq}}^{GOS}(s) = E[e^{-s\gamma}]$, we get

$$M_{\gamma_{eq}}^{GOS}(s) = M_{sr}M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1} \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} \exp\left(\frac{(n_1+k_1)(n_2+k_2)C}{2\bar{\gamma}_2(n_1+k_1+s\bar{\gamma}_1)}\right) (-1)^\varphi \left[\frac{W_{-1,1/2}(\sigma)}{(n_2+k_2)(n_1+k_1+s\bar{\gamma}_1)} + \sqrt{\frac{C(n_1+n_2+s\bar{\gamma}_1)^{-1}}{(n_1+k_1)(n_2+k_2)\bar{\gamma}_2}} \right] W_{-1/2,0}(\sigma) \quad (6)$$

where $\sigma = \frac{(n_1+k_1)(n_2+k_2)C}{\bar{\gamma}_2(n_1+k_1+s\bar{\gamma}_1)}$, and $\varphi = k_1 + k_2$. We have used [15, 6.643-3] to arrive at the final expression, and $W_{\dots}(\cdot)$ is Whittaker function defined in [15, 9.22]. When $M_s = M_r = M_d = 1$, (6) is equivalent to [12, eqn. 12].

Fig. 4: Average End-end SNR for General Order Antenna Selection in MIMO Relay for same total number of antennas

C. General Moments

In this subsection, we characterize the general moments of the end-to-end SNR γ_{eq} . The general moments are important measure matrix, which can be used to obtain the end-to-end SNR γ_{eq} , variance, and amount of fading (AoF). By definition, the generalized moments of γ_{eq} can be given by,

$$\mu_n = n \int_0^\infty \gamma^{n-1} [1 - F_{\gamma_{eq}}^{GOS}(\gamma)] d\gamma \quad (7)$$

To this end, substituting the CDF expression given by (22) and with the help of [15, 6.643-3], the closed-form expression for the n -th moments of γ_{th} can be expressed as follows:

$$\mu_n = M_{sr}M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1} \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} (-1)^{k_1+k_2} \left(\frac{\bar{\gamma}_1}{n_1+k_1}\right)^{n+1} \frac{n(n+2)(n!)^2}{(n_1+k_1)(n_2+k_2)} \exp\left(\frac{C(n_2+k_2)}{2\bar{\gamma}_2}\right) W_{-(n+1),1/2}\left(\frac{(n_2+k_2)C}{\bar{\gamma}_2}\right) \quad (8)$$

As a direct application, the average end-to-end SNR can be obtained as $n = 1$, and the AoF, which quantifies of fading severity, can be obtained by,

$$AoF = \frac{E[\gamma_{eq}^2] - \{E[\gamma_{eq}]\}^2}{\{E[\gamma_{eq}]\}^2} = \frac{\mu_2}{\mu_1^2} - 1 \quad (9)$$

Fig. 4 shows the average end-end SNR of the system for the highest order antenna selection at both hops again for a fixed number of antennas in the system. The evenly distributed system offers the highest average SNR, whereas the system with multi-antenna relay performs poorly.

IV. SNR STATISTICS WITH DIRECT LINK

In this section, we obtain SNR statistics for GOAS in MIMO Relay taking account of the direct link between S and D . The relayed and the direct links are assumed independent of each other, and the received signals at D can be processed either using Selection-Combining (SC) or Maximal-Ratio-Combining (MRC). Note that the antenna selection at the source for both the relayed and the direct is not possible at the same time unless there are separate antenna sets one each for the relayed and the direct links. We make such assumption and restrict ourselves to go in details of such scenarios. The motivation is to compare the performance gain when the direct link exists.

A. SC at Destination

When the largest signal is selected between the relayed and the direct signals, i.e. selection combining (SC) at the destination, we can obtain a closed-form expression for the CDF $F_{\gamma_{tot}}^{SC}(\gamma)$. Since the output instantaneous SNR of γ_{tot} is given by

$$\gamma_{tot} = \max\{\gamma_{sd}, \gamma_{eq}\} \quad (10)$$

Thus, we can write the CDF of selected-branch at the destination terminal as

$$F_{\gamma_{tot}}^{SC}(\gamma) = F_{\gamma_{sd}}^{GOS}(\gamma)F_{\gamma_{eq}}^{GOS}(\gamma) \quad (11)$$

and the PDF $f_{\gamma_{tot}}^{SC}(\gamma)$ can be obtained as

$$\begin{aligned} f_{\gamma_{tot}}^{SC}(\gamma) &= \frac{d}{d\gamma} F_{\gamma_{tot}}^{SC}(\gamma) \\ &= f_{\gamma_{eq}}^{GOS}(\gamma)F_{\gamma_{sd}}^{GOS}(\gamma) + F_{\gamma_{eq}}^{GOS}(\gamma)f_{\gamma_{sd}}^{GOS}(\gamma) \end{aligned} \quad (12)$$

B. MRC at Destination

When the signals from the direct and the relayed links are combined coherently at the destination (MRC), i.e.

$$\gamma_{tot} = \gamma_{sd} + \gamma_{eq} \quad (13)$$

it is difficult to obtain a closed-form expression. The conventional approach is based on MGFs. When two independent random variables (RVs) are coherently combined (added), the MGF of the resultant RV is equal to the product of MGFs of the individual RVs, i.e.

$$M_{\gamma_{tot}}^{MRC}(s) = M_{\gamma_{eq}}^{GOS}(s)M_{\gamma_{sd}}^{GOS}(s) \quad (14)$$

where $M_{\gamma_{eq}}^{GOS}(s)$ is given the expression in (6), and $M_{\gamma_{sd}}^{GOS}(s)$ is given by

$$\begin{aligned} M_{\gamma_{sd}}^{GOS}(s) &= M_{sd} \binom{M_{sd}-1}{n_3-1} \sum_{k_3=0}^{M_{sd}-n_3} \binom{M_{sd}-n_3}{k_3} \\ &\quad \frac{(-1)^{k_3}}{n_3 + k_3 + s\bar{\gamma}_3} \end{aligned} \quad (15)$$

and $M_{sd} = M_s M_d$. The inverse Laplace transform of the MGF gives the PDF.

Fig. 5: A MIMO Dual-Hop System with Multiple Relays.

V. SNR STATISTICS WITH MULTIPLE RELAYS

The analysis presented in previous section can be extended to the scenario where there are multiple relays to relay the signal from the source to the destination. We assume that the multi-antenna relays have same number of antenna M_r and subject to similar fading scenario (i.i.d. Rayleigh fading among antennas). The highest relayed signal is selected (Selection Relaying, SR) for demodulation at the destination, i.e. $\gamma_{tot} = \max\{\gamma_{eq_1}, \dots, \gamma_{eq_i}, \dots, \gamma_{eq_N}\}$. The SNR CDF $F_{\gamma_{tot}}^{SR}(\gamma)$ can then be given as,

$$\begin{aligned} F_{\gamma_{tot}}^{SR}(\gamma) &= [F_{\gamma_{eq}}^{GOS}(\gamma)]^N \\ &= \sum_{i=0}^N \binom{N}{i} \Omega \left[\sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} \right. \\ &\quad \left. \frac{(-1)^{k_1+k_2+1}}{e^{(n_1+k_1)\frac{\gamma}{\bar{\gamma}_1}}} \sqrt{\frac{C\gamma}{(n_1+k_1)(n_2+k_2)\bar{\gamma}_1\bar{\gamma}_2}} K_1(\lambda) \right]^i \end{aligned} \quad (16)$$

where N is the number of relays and $\Omega = [2M_{sr}M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1}]^i$. The above expression is applicable when the relayed signals are independent to each other. Same set of ordered antennas (i.e. n_1 and n_2 are same for all the relayed signals) is selected in all the relays to facilitate simpler expression.

One scenario of practical interest could be when the best link is selected for all relayed signals, i.e. $n_1 = n_2 = 1$. Above expression becomes,

$$\begin{aligned} F_{\gamma_{tot}}^{SR}(\gamma) &= \sum_{i=0}^N \binom{N}{i} \left[2M_{sr}M_{rd} \sum_{k_1=0}^{M_{sr}-1} \sum_{k_2=0}^{M_{rd}-1} \binom{M_{sr}-1}{k_1} \right. \\ &\quad \left. \binom{M_{rd}-1}{k_2} \frac{(-1)^{k_1+k_2+1}}{e^{(k_1+1)\frac{\gamma}{\bar{\gamma}_1}}} \sqrt{\frac{C\gamma}{(k_1+1)(k_2+1)\bar{\gamma}_1\bar{\gamma}_2}} K_1(\lambda) \right]^i \end{aligned} \quad (17)$$

The MRC of all the relayed signals provides the optimum performance, but would call for more complicated derivations. Above analysis can be further applied to the case which takes account of the presence of both the direct link and the multiple relays, and again for the selection combining at the destination the derivation is straightforward.

VI. PERFORMANCE ANALYSIS

In this section, we analyze the impact of order selection on the system performance in terms of outage probability. The modified bessel function $K_1(\cdot)$ and the Whittaker function $W_{\dots}(\cdot)$ in the final expressions were evaluated in MATLAB. The gain G is fixed through the constant $C = \frac{\gamma_1}{e^{1/\gamma_1} E_1(1/\gamma_1)}$ [12, eq. 16], where E_1 is Euler integral.

Fig. 6 shows the outage probability for a fixed number of antennas in the system. Balanced system ($M_s = M_r = M_d = 2$) is better as it provides superior diversity gain for both the direct and the relayed links, unlike ($M_s = M_d = 1, M_r = 4$) system which greatly suffers from the SISO direct link. However, the presence of direct link improves the performance even in the second scenario.

Next we illustrate the impact of antenna order selection on the performance loss in Fig. 7. Obviously, lower order antenna selection in any link will degrade the end-end performance. Moreover, the relay system are more prone to performance loss when the first link is under severe fading or not that good. As can be seen from the figure, as the antenna order at the first hop decreases ($n_1 = 1, 2, 3$) the outage shifts towards right indicating higher loss.

The last Fig. 8 shows the outage probability for number of identical multi-antenna relays willing to assist relaying information to the destination. For simplicity, we assumed identical number of antennas across the terminals and same antenna order selection for all the hops. Best relayed signal is chosen at the destination for the demodulation. More the relays, lower the outage.

VII. CONCLUSION

In this contribution, we have presented new general expressions of SNR statistics for general order antenna selection in a MIMO dual-hop amplify-forward relay network. The antenna selection is applied at both hops. The SNR statistics such as CDF, PDF, MGF and General Moments are derived, and then used to analyze the system with the direct link and the multiple relays. Numerical results are given to analyze the system outage behaviour and its dependence on the antenna order selection. The results indicate the usefulness of the derived expressions to accurately quantify the metrics such as average SNR loss due to selection of lower order antenna (instead of highest order or best antenna). The expressions specialize to earlier results, and were verified by computer simulation.

APPENDIX I

PROOF OF THEOREM 1

In this appendix, we derive the c.d.f. expression of random variable $\gamma_{eq} = \frac{\gamma_1 \gamma_2}{C + \gamma_2}$. Before we proceed to derive the CDF, we need the following lemma on the PDF of order statistic.

Fig. 6: Comparison of Outage Probability for Best Antenna Selection in MIMO Relay for same total number of antennas, with Direct Link; '*' showing for $M_s = M_r = M_d = 2$ and '+' for $M_s = 1, M_r = 4, M_d = 1$

Fig. 7: Impact of General Order Antenna Selection on Outage Probability in MIMO Relay with Direct Link

Lemma 1: Let $\Gamma_1, \Gamma_2, \dots, \Gamma_M$ be M independent and identical random variables, and arranged in decreasing order denoted by $\Gamma_{(1)}, \Gamma_{(2)}, \dots, \Gamma_{(M)}$ where $\Gamma_{(1)}$ corresponding to the highest order statistic (largest of the Γ_i 's). If $f(\gamma)$ and $F(\gamma)$ are the PDF and the CDF of Γ_i 's respectively, the PDF of n^{th} order statistic $\Gamma_{(n)}$ is given by,

$$f_{\Gamma_{(n)}}(\gamma) = M \binom{M-1}{n-1} [F(\gamma)]^{M-n} [1 - F(\gamma)]^{n-1} f(\gamma) \quad (18)$$

For i.i.d. Rayleigh faded variable, the PDF and the CDF of n^{th} order statistics are given by,

$$f_{\gamma_n}(\gamma) = M \binom{M-1}{n-1} \sum_{k=0}^{M-n} \binom{M-n}{k} \frac{(-1)^k}{\bar{\gamma}} \times e^{-(n+k)\frac{\gamma}{\bar{\gamma}}} \quad (19)$$

Fig. 8: Impact of General Order Antenna Selection on Outage Probability in MIMO Selection Relaying; $N = 1, 2, 3, 4$

and,

$$F_{\gamma_n}(\gamma) = M \binom{M-1}{n-1} \sum_{k=0}^{M-n} \binom{M-n}{k} \frac{(-1)^k}{n+k} \times \left[1 - e^{-(n+k)\frac{\gamma}{\gamma}} \right] \quad (20)$$

To this end, from the definition of CDF of γ_{eq} , we have

$$F_{\gamma_{eq}}^{GOS}(\gamma) = \Pr(\gamma_{eq} < x\gamma_{eq}) = \Pr\left(\gamma_1 < \left(1 + \frac{C}{\gamma_2}\right)\gamma\right). \quad (21)$$

Conditioned on γ_1 , and γ_2 , the CDF of γ_{eq} can be expressed

$$\begin{aligned} F_{\gamma_{eq}}^{GOS}(\gamma) &= \int_0^\infty \Pr\left(\gamma_1 < \left(1 + \frac{C}{\gamma_2}\right)\gamma\right) f_{\gamma_{(n_2)}}(\gamma_2) d\gamma_2 \\ &= 1 - M_{sr} M_{rd} \binom{M_{sr}-1}{n_1-1} \binom{M_{rd}-1}{n_2-1} \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \\ &\quad \binom{M_{sr}-n_1}{k_1} \binom{M_{rd}-n_2}{k_2} \frac{(-1)^{k_1+k_2}}{\gamma_2(n_1+k_1)} \\ &\quad e^{-(n_1+k_1)\frac{\gamma}{\gamma_1}} \underbrace{\int_0^\infty e^{-(n_1+k_1)\frac{\gamma C}{\gamma_1 \gamma_2} - (n_2+k_2)\frac{\gamma_2}{\gamma_2}} d\gamma_2}_{\mathcal{I}_1} \end{aligned} \quad (22)$$

To this end, the desired result can be obtained after some simple algebraic manipulations with the help of formula [15, 3.324-1].

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