RESEARCH ARTICLE

Capacity limits of spectrum-sharing systems over hyper-fading channels

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ABSTRACT

Cognitive radio (CR) with spectrum-sharing feature is a promising technique to address the spectrum under-utilization problem in dynamically changing environments. In this paper, the achievable capacity gain of spectrum-sharing systems over dynamic fading environments is studied. To perform a general analysis, a theoretical fading model called hyper-fading model that is suitable to the dynamic nature of CR channel is proposed. Closed-form expressions of probability density function (PDF) and cumulative density function (CDF) of the signal-to-noise ratio (SNR) for secondary users (SUs) in spectrum-sharing systems are derived. In addition, the capacity gains achievable with spectrum-sharing systems in high and low power regions are obtained. The effects of different fading figures, average fading powers, interference temperatures, peak powers of secondary transmitters, and numbers of SUs on the achievable capacity are investigated. The analytical and simulation results show that the fading figure of the channel between SUs and primary base-station (PBS), which describes the diversity of the channel, does not contribute significantly to the system performance gain. Copyright © 2011 John Wiley & Sons, Ltd.

KEYWORDS

spectrum-sharing system; hyper-Nakagami fading; mixture model; multiuser diversity; cognitive radio; achievable capacity; hyper fading

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1. INTRODUCTION

The frequency spectrum is one of the most precious and limited resources in wireless communication systems. Therefore, regulatory agencies exclusively allocate each band in spectrum to a specific user and guarantee that this licensed user will be protected from any interference. On the other hand, the recent spectrum measurement campaigns around the world, such as those given in References [1–3], show that the spectrum is under-utilized. Under these conservative frequency allocation policy, the spectrum becomes scarce especially with the emergence of new wireless services and technologies [4–8].

Cognitive radio (CR) is a promising technology to improve the efficiency of spectrum utilization. The two main spectrum access approaches proposed in the literature are opportunistic spectrum access [7] and spectrum sharing [8]. In the first approach, secondary users (SUs) are allowed to use the licensed bands when the corresponding primary users (PUs) are idle. In the second approach, SUs are allowed to utilize the spectrum licensed to PUs only if the interference caused by SUs is below a threshold, which is called interference temperature [9]. Note that the spectrum sharing is a more aggressive method than the first one; hence, recently, it has attracted considerable attention.

The capacity of SU in a spectrum-sharing system is derived in Reference [10] over non-fading additive white Gaussian noise (AWGN) channels under received power constraint. The effects of multi-user diversity on the capacity of a spectrum-sharing system where multiple SUs utilize the licensed spectrum are investigated in Reference [11]. In the multi-user diversity (gain) technique, the aim is to have the best channel quality for the communication system. This method shows that the system has maximum throughput [12] in non-spectrum-sharing systems. There have been numerous studies on the effects of multi-user diversity on non-spectrum-sharing systems [12–17]. In spectrum-sharing systems, this effect has been actively studied in References [8,10,18]. The limits on the channel capacity...
for a CR system has been recently studied in References [19,20]. In Reference [8], the capacity of a spectrum-sharing system is analyzed considering symmetric fading models (Rayleigh and Nakagami) in the presence of multiple PUs. The work in Reference [8] is extended in Reference [21] by studying the channel capacity limits of spectrum-sharing systems in asymmetric fading environments. This is where SU transmitter–PU receiver path and SU transmitter–SU receiver path could experience different fading types and link powers due to path length or shadowing.

The previous works motivate us to develop a theoretical fading model that can be used to perform a unified analysis for CR spectrum-sharing systems. Due to the highly dynamic nature of propagation environments, several single-fading models are employed in the literature for the analysis of CR spectrum-sharing systems. However, considering practical scenarios, it would be more efficient and convenient to use a generic fading model, which can be degenerated onto widely used single-fading models with appropriate selection of parameters. In this paper, the proposed generic fading model, which is termed hyper-Nakagami-$m$ fading, represents several widely encountered propagation scenarios such as line-of-sight (LOS)/non-line-of-sight (NLOS) environments and fixed/mobile transmissions. Additionally, instantaneous and average power capacity calculations can also be carried out with the proposed generic model properly. In the light of the analysis presented for the proposed method, the capacity of SU in a spectrum-sharing system is studied under SU transmit power and interference temperature constraints. Numerical results along with relevant discussions are provided.

The rest of the paper is organized as follows. In Section 2, the system model is presented. This is followed by the statistical background on the proposed hyper-fading model in Section 3. In Section 4, capacity of spectrum-sharing systems is derived for both high and low power regions. In Section 5, the analytical and numerical results are presented. Finally, the conclusions are provided in Section 6.

2. SYSTEM MODEL

The system model in Reference [11] is adopted in this paper. In Reference [11], a symmetric fading channel is considered where SU transmitter–PU receiver (interference channel) and SU transmitter–SU receiver (desired channel) channel gains are assumed to be independent and identically distributed exponential random variables (RVs) with unit mean in independent Rayleigh fading channels. However, in practice, these channels can be dynamic, as a result, the fading conditions and link powers can be time-varying. Therefore, in this paper, we assume that both channels are independent and non-identically distributed hyper-Nakagami-$m$ fading RVs that might represent any type of fading environments.

The system model is shown in Figure 1, where $\phi_i$ and $\psi_i$ are the interference and desired channel gains, respectively, and $N_s$ stands for the number of secondary transmitters. In spectrum-sharing systems, the interference power levels caused by the SU-transmitters at the primary receivers must not be larger than some predefined value $Q$, referred to as the interference temperature. It is assumed that the perfect information of interference channels, $\phi_i$, is available at SU-transmitters. The SU-transmitters can obtain this information, which is also termed as channel side information (CSI), through various ways such as direct feedback from PU-receiver [8] or from a mediate band manager between the PU-receiver and SU-transmitters [11,22]. In addition, the opportunist SU selection strategy is employed herein [11], where SU receiver selects the SU with the maximum signal-to-noise ratio (SNR) value.

Note also that the interference from PUs is not considered in this analysis and the detailed analysis of the operation and protocol between the PU-receiver and SU-transmitters has been already studied in References [8,11,22]. The interference from PUs can be considered as an additive disturbance which can be modeled as a colored noise source in primary base-station (PBS). Recalling that basic transmitter–receiver chain, such disturbances as noise and interference are assumed to be added into the signal after the transmit signal is convolved with the channel impulse response. Since the focus of this study is to propose a model for fading statistics of CR channels, such additive disturbances would not change the analysis of fading statistics. However, as future work, it would be very interesting to observe how the proposed model performs in the presence of interference plus noise from the system capacity perspective. For the rest of the paper, we will refer to the primary receiver as the PBS and to the secondary receiver as the secondary base-station (SBS).

3. STATISTICAL BACKGROUND

Radio wave propagation in wireless cognitive channels is a complex phenomenon characterized by three nearly independent phenomena, which are the path-loss variance with distance, shadowing (or long-term fading), and multipath...
(or short-term) fading. Except path-loss variance, which is only distance dependent, such various effects as fading, reflection, refraction, scattering, and shadowing are related to the other two phenomena. Therefore, the majority of the studies in the literature are considerably devoted to characterizing these effects through the medium of statistical models, which are based on measurements performed for a specific channel environment. Furthermore, these three nearly independent phenomena change according to the communication environment, carrier-frequency, and bandwidth. There are numerous channel fading models which have been proposed in the literature to statistically model these phenomena with envelope distributions, regarding pretest evaluation of wireless communications systems in general, and of fading mitigation techniques in particular. Briefly, several statistical distributions have been proposed for channel fading modeling under short-term and long-term fading conditions due to the existence of a great variety of fading environments. For instance, short-term fading models include the well-known Rayleigh, Weibull, Rice, and Nakagami-m [23–27] distributions, while long-term fading models are modeled by the well-known log-normal distribution [28,29].

In CR communications, the fading conditions are subject to change according to the environment $\xi$, in each of which the fading conditions are indexed by the carrier frequency $f_c$, the bandwidth $B$, and the position $X$ such that $\xi \sim \xi(f_c, B, X)$. The fading can be expressed as follows.

### 3.1. Definition: hyper-Nakagami-m fading distribution

Consider a RV $R$ which follows the hyper-Nakagami-m fading envelope distribution with a probability density function (PDF) given by

$$f_R(r) = \sum_{k=0}^{N} \frac{2^{m_k}}{\Gamma(m_k)} \left( \frac{m_k}{\Omega_k} \right)^m r^{m_k-1} \exp \left( -\frac{m_k r^2}{\Omega_k} \right)$$

where for $1 \leq k \leq N$, the parameters $m_k \geq 0.5$, $\Omega_k > 0$, and $0 \leq \xi_k \leq 1$ are the fading figure, the average power, and the accruing factor of the $k$th fading environment, respectively. The accruing factors due to $N$ possible fading environments satisfy the condition:

$$\sum_{k=1}^{N} \xi_k = 1$$

If the users are subject to the hyper-Nakagami-m fading, then the distribution of the instantaneous SNR, $\gamma \triangleq R^2/N_0$ in AWGN channels can be directly expressed in terms of average SNR, $\overline{\gamma} \triangleq E\{ R^2 \}/N_0$ with $E\{ \}$ denoting the expectation operator and $N_0$ representing the power of AWGN noise. The SNR variable $\gamma$ is assumed to be hyper-Gamma distributed.

### 3.2. Definition: hyper-Gamma fading power distribution

The expression of hyper-Gamma distribution of RV $\gamma$ is given by

$$f_{\gamma}(r) = \sum_{k=0}^{N} \frac{\xi_k}{\Gamma(m_k)} \left( \frac{m_k}{\Omega_k} \right)^{m_k} r^{m_k-1} \exp \left( -\frac{m_k r^2}{\Omega_k} \right)$$

where $\Omega_k > 0$ is the average SNR value.

It is well known that in a communication system the obstructions between the transmitter and the receiver make the system to undergo different types of channel fading such as Rayleigh and Nakagami-m. Moreover, basic effects, such as the speed and the motion direction of the user, cause the communication channel fading conditions to change even for very short periods of time. In spite of the aforementioned dynamic nature of fading channels, single fading models are mainly used for the analysis of CR systems. However, it is more realistic to use mixture models, which are the weighted combination of different fading distributions [30]. Such a dynamic fading phenomenon is more pronounced in CR channels since the utilized $f_c$ and $B$ can change in addition to the changes in the environment. Therefore, mixture fading models are more suitable for the analysis of CR channels. As a result, in our system, we use the hyper-Nakagami-m fading model, in which the CR communication system might undergo a different number of channel fading models such as Rayleigh, Gaussian, or Nakagami-3. Let us consider a case where the CR system undergoes four different fading channels ($N=4$). Assume that the CR system experiences first one-sided Gaussian fading ($m_1=0.5$) with accruing factor of $\xi_1=0.1361$, then Rayleigh fading ($m_2=1$) with accruing factor of $\xi_2=0.3319$, and then Nakagami-m fading ($m_3=2$) with accruing factor of $\xi_3=0.38$, and finally Nakagami-m fading ($m_4=3$) with accruing factor of $\xi_4=0.152$. Note that based on the simulations, we found that the accruing factors for Rayleigh (i.e., $m=1$) and for Nakagami-m (i.e., $m=2$) are higher than the values corresponding to other fading environments such as the one-sided Gaussian fading (i.e., $m=0.5$). The above case is simulated in Figure 2, which shows the PDF of power for different channel fading models $\xi_k$ with $N=4$ different environments. It is assumed that the average power for each type of fading is unity. Furthermore, it is well known that the one-sided Gaussian fading and Rayleigh fading are special cases of the Nakagami-m fading.

### 4. Capacity of Spectrum-Sharing System

In this system, there are two assumptions for the SU-transmitter power. First, it needs to be within its allowable maximum power constraints. Second, it is not allowed to be higher than the predefined interference temperature value $Q$. 

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in order not to cause any interference on PU-receiver. When the interference power level \( P \) caused by SU-transmitter at the PU-receiver achieves a value larger than \( Q \), an adaptive scheme is used to adjust its value. Therefore, the transmit power of the \( i \)th SU is given by

\[
P_i = \begin{cases} P, & \psi_i \leq \frac{Q}{P} \\ \frac{Q}{P}, & \text{otherwise} \end{cases} \tag{4}
\]

where \( \psi_i \) is a hyper-Gamma RV, which represents the distribution for the fading power of the channel between the \( i \)th SU and the PBS, and \( P \) and \( Q \) are the peak power of the SUs and the allowable interference temperature level at the PBS, respectively. Then, the adjusted power \( P_i \) is used for sending data from the \( i \)th SU to a target SBS. Thus, the received SNR at the target SBS is given by

\[
S_i = \begin{cases} \psi_i P, & \psi_i \leq \frac{Q}{P} \\ \frac{Q}{P}, & \text{otherwise} \end{cases} \tag{5}
\]

where both \( \psi_i \) and \( \psi_i \) are modeled as mutually independent and non-identically distributed two hyper-Gamma fading power distributions whose PDFs are given, respectively, by

\[
f_{\psi,\phi}(r) = \sum_{l=0}^{N} \frac{\xi_{\psi,l} \xi_{\phi,l}}{\Gamma(m_{\psi,l}) \Gamma(m_{\phi,l})} \left( \frac{m_{\psi,l}}{\tau_{\psi,l}} \right)^{m_{\psi,l}-1} e^{-\frac{m_{\psi,l}}{\tau_{\psi,l}}} r^{m_{\phi,l}-1} e^{-\frac{m_{\phi,l}}{\tau_{\phi,l}}} \tag{6}
\]

\[
f_{\psi,\phi}(r) = \sum_{k=0}^{N} \frac{\xi_{\psi,k} \xi_{\phi,k}}{\Gamma(m_{\psi,k}) \Gamma(m_{\phi,k})} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} r^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} \tag{7}
\]

The conditional PDF of the received SNR at the target SBS, \( f_S(r|\psi_i) \), is given by

\[
f_S(r|\psi_i) = \begin{cases} \sum_{k=0}^{N} \frac{\xi_{\psi,k} \xi_{\phi,k}}{\Gamma(m_{\psi,k}) \Gamma(m_{\phi,k})} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} r^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} & \psi_i \leq \frac{Q}{P} \\ \sum_{k=0}^{N} \frac{\xi_{\psi,k} \xi_{\phi,k}}{\Gamma(m_{\psi,k}) \Gamma(m_{\phi,k})} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} r^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} & \psi_i > \frac{Q}{P} \end{cases} \tag{8}
\]

Then, the PDF of the received SNR at the target SBS can be obtained as

\[
f_S(r) = \int_{0}^{\infty} f_S(r|\psi_i) f_S(\psi_i) d\psi_i \tag{9}
\]

Considering the limits of summation in both Equations (6) and (8) along with the integration interval of Equation (9), \( f_S(r) \) is obtained by manipulating the cross terms and given by in a more simplified form as

\[
f_S(r) = \sum_{k=0}^{N} \sum_{l=0}^{N} \frac{\xi_{\psi,k} \xi_{\phi,l}}{\Gamma(m_{\psi,k}) \Gamma(m_{\phi,l})} \left( \frac{m_{\psi,l}}{\tau_{\psi,l}} \right)^{m_{\psi,l}-1} e^{-\frac{m_{\psi,l}}{\tau_{\psi,l}}} r^{m_{\phi,l}-1} e^{-\frac{m_{\phi,l}}{\tau_{\phi,l}}} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} \left( \frac{m_{\phi,k}}{\tau_{\phi,k}} \right)^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} + \left( \frac{m_{\psi,l}}{\tau_{\psi,l}} \right)^{m_{\psi,l}-1} e^{-\frac{m_{\psi,l}}{\tau_{\psi,l}}} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} \left( \frac{m_{\phi,l}}{\tau_{\phi,l}} \right)^{m_{\phi,l}-1} e^{-\frac{m_{\phi,l}}{\tau_{\phi,l}}} \left( \frac{m_{\phi,k}}{\tau_{\phi,k}} \right)^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} \right) \tag{10}
\]

where \( \Gamma(x, y) = \int_{0}^{\infty} t^{x-1} e^{-t} \frac{dt}{y} \) is the incomplete gamma function. Then, the cumulative density function (CDF) of \( S_i \) is obtained by using

\[
F_S(r) = \int_{0}^{r} f_S(r) dr \tag{11}
\]

By plugging Equation (10) into Equation (11) and carrying out the integral, the CDF \( F_S(r) \) is given by

\[
F_S(r) = \sum_{k=0}^{N} \sum_{l=0}^{N} \frac{\xi_{\psi,k} \xi_{\phi,l}}{\Gamma(m_{\psi,k}) \Gamma(m_{\phi,l})} \left( \frac{m_{\psi,l}}{\tau_{\psi,l}} \right)^{m_{\psi,l}-1} e^{-\frac{m_{\psi,l}}{\tau_{\psi,l}}} r^{m_{\phi,l}-1} e^{-\frac{m_{\phi,l}}{\tau_{\phi,l}}} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} r^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}} + \left( \frac{m_{\psi,l}}{\tau_{\psi,l}} \right)^{m_{\psi,l}-1} e^{-\frac{m_{\psi,l}}{\tau_{\psi,l}}} \left( \frac{m_{\psi,k}}{\tau_{\psi,k}} \right)^{m_{\psi,k}-1} e^{-\frac{m_{\psi,k}}{\tau_{\psi,k}}} \left( \frac{m_{\phi,l}}{\tau_{\phi,l}} \right)^{m_{\phi,l}-1} e^{-\frac{m_{\phi,l}}{\tau_{\phi,l}}} \left( \frac{m_{\phi,k}}{\tau_{\phi,k}} \right)^{m_{\phi,k}-1} e^{-\frac{m_{\phi,k}}{\tau_{\phi,k}}}ight) \tag{12}
\]
SNR at the target SBS, disappears. Therefore, the conditional PDF of the received SNR, which is the CDF of the maximum received SNR evaluated and using Equation (9), the corresponding CDF is obtained

$$f_{S_{\text{max, out}}}(r_{\text{th}}) = \int_{0}^{r_{\text{th}}} f_{S_{\text{max}}}(r)dr$$

where the integration is evaluated by the Gauss–Laguerre quadrature rule [31]† and \(\Xi(u)\) is given by

$$\Xi(u) = \frac{u^{m_{\psi,k} - 1} \Gamma(m_{\psi,k} + m_{\varphi,l}, \frac{m_{\psi,k} Q}{P})}{(1 + m_{\psi,k} m_{\varphi,l} + m_{\psi,k} Q) \Gamma(m_{\psi,k} + m_{\varphi,l} + 1)}$$

(13)

Consequently, the SBS selects an SU transmitter with the best channel quality among the \(N_s\) (number of SUs) SNR values of SU transmitters. The received SNR of the selected SU \(S_{\text{max}}\) is obtained as

$$S_{\text{max}} = \max_{1 \leq s \leq N_s} S_s$$

(14)

Assuming that every user is equally faded, then the PDF of \(S_{\text{max}}\) is given by [11]

$$f_{S_{\text{max}}}(r) = N_s f_S(r) F_S(r)^{N_s - 1}$$

(15)

and overall average achievable capacity is obtained by

$$C = E[\log_2(1 + S_{\text{max}})]$$

$$= \int_{0}^{\infty} \log_2(1 + S_{\text{max}}) f_{S_{\text{max}}}(r)dr$$

(16)

Since outage probability is a common performance metric, here we consider the outage probability of \(S_{\text{max}}\) for different scenarios in terms of the following:

$$f_{S_{\text{max, out}}}(r_{\text{th}}) = \int_{0}^{r_{\text{th}}} f_{S_{\text{max}}}(r)dr$$

(17)

which is the CDF of the maximum received SNR evaluated at the outage threshold \(r_{\text{th}}\) [dB]. The results for outage probability with respect to number of SUs are given in Figure 3. As expected, the probability of outage saturates as \(r_{\text{th}}\) increases which implies that the received signal power is weakening. For a specific outage threshold value, increasing the number of SUs results in decrease in the probability of outage, which stems from the effects of multiuser diversity.

**4.1. Low power region analysis**

If \(P \ll Q\) then the effect of interference temperature level \(Q\) disappears. Therefore, the conditional PDF of the received SNR at the target SBS, \(f_S(r|\psi)\) is given by

$$f_S(r|\psi) = \sum_{k=0}^{N_s} \frac{\xi_{\psi,k} e^{m_{\psi,k} - 1}}{\Gamma(m_{\psi,k})} \left(\frac{m_{\psi,k}}{\gamma_{\psi,k} Q}\right)^{m_{\psi,k}} e^{-\frac{m_{\psi,k}}{\gamma_{\psi,k} Q} r}$$

(18)

Then, the PDF of the received SNR at the target SBS is approximated as

$$f_S(r) = \sum_{k=0}^{N_s} \frac{\xi_{\psi,k} e^{m_{\psi,k} - 1}}{\Gamma(m_{\psi,k})} \left(\frac{m_{\psi,k}}{\gamma_{\psi,k} Q}\right)^{m_{\psi,k}} e^{-\frac{m_{\psi,k}}{\gamma_{\psi,k} Q} r}$$

(19)

and using Equation (9), the corresponding CDF is obtained as

$$F_S(r) = \sum_{k=0}^{N_s} \frac{1}{\Gamma(m_{\psi,k})} \left(\frac{m_{\psi,k}}{\gamma_{\psi,k} Q}\right)^{m_{\psi,k}} e^{-\frac{m_{\psi,k}}{\gamma_{\psi,k} Q} r}$$

(20)

The received SNR of the selected SU \(S_{\text{max}}\) is obtained using Equation (14). Assuming that every user is equally faded, then the PDF of \(S_{\text{max}}\) and overall average achievable capacity is obtained using Equations (15) and (16), respectively.

**4.2. High power region analysis**

If \(P \gg Q\), then the conditional PDF of the received SNR at the target SBS, \(f_S(r|\psi)\) is given by

$$f_S(r|\psi) = \sum_{k=0}^{N_s} \frac{\xi_{\psi,k} e^{m_{\psi,k} - 1}}{\Gamma(m_{\psi,k})} \left(\frac{m_{\psi,k}}{\gamma_{\psi,k} Q}\right)^{m_{\psi,k}} e^{-\frac{m_{\psi,k}}{\gamma_{\psi,k} Q} r}$$

(21)

Then, using Equation (9), the PDF of the received SNR at the target SBS is obtained as

† See Eq. (25.4.45).
\[ f_S(r) = \sum_{k=0}^{N} \sum_{l=0}^{N} \frac{\xi_{\phi,k} \xi_{\phi,l}}{\Gamma(m_{\phi,k}) \Gamma(m_{\phi,l})} \left( \frac{m_{\phi,k} T_{\phi,l}^{-1}}{\sum m_{\phi,k} T_{\phi,l}^{-1}} \right)^{m_{\phi,k}} \]
\[ \times r^{m_{\phi,k}-1} \frac{\Gamma(m_{\phi,k} + m_{\phi,l})}{\Gamma(m_{\phi,k} + m_{\phi,l})} \left( 1 + \frac{m_{\phi,k} T_{\phi,l}^{-1}}{\sum m_{\phi,k} T_{\phi,l}^{-1}} \right)^{-m_{\phi,k} + m_{\phi,l}} \] (22)

Using Equation (11), the CDF of \( S \) is obtained as
\[ F_S(r) = \sum_{k=0}^{N} \sum_{l=0}^{N} \frac{\xi_{\phi,k} \xi_{\phi,l}}{\Gamma(m_{\phi,k}) \Gamma(m_{\phi,l})} \left( \frac{m_{\phi,k} T_{\phi,l}^{-1}}{\sum m_{\phi,k} T_{\phi,l}^{-1}} \right)^{m_{\phi,k}} \]
\[ \times \int_{0}^{\infty} u^{m_{\phi,k}-1} \left( 1 + \frac{m_{\phi,k} T_{\phi,l}^{-1}}{\sum m_{\phi,k} T_{\phi,l}^{-1}} \right)^{-m_{\phi,k} + m_{\phi,l}} du \] (23)

where the integration can be easily evaluated by Gauss-Laguerre quadrature rule [31]. Consequently, the SBS selects a SU using Equation (14). Assuming that every user is equally faded, then the PDF of \( S_m \) and overall average achievable capacity are obtained using Equations (15) and (16), respectively.

5. RESULTS

The effect of peak power of secondary transmitters on the average capacity is investigated under different values for the number of environments \( N \) and interference temperature values \( Q \) in Figure 4. These simulations assume the same environment accruing factors \( \xi \), the fading figure values \( m \), and average fading power between the SUs and the SBS as given in Figure 2. In addition, the comparison of the hyper-Nakagami-\( m \) fading channel model with the Rayleigh fading channel is done. It is shown that the average capacity increases as the peak power of the secondary transmitters increases for both Rayleigh and Nakagami-\( m \) fading as expected. However, unlike the non-spectrum-sharing systems the average capacity is here saturated after a certain value of peak power because of the spectrum-sharing system opportunistic user selection algorithm [11]. It is seen from the figures that the analytical results agree well with the simulation results.

In Figure 5, the effect of interference temperature on the average capacity is investigated in detail under different values of the number of environments \( N \) and peak power of secondary transmitters \( P \), with the same environment accruing factors \( \xi \), the fading figure values \( m \), and the average fading power between the SUs and the SBS as given for Figure 2. The comparison of the hyper-Nakagami-\( m \) fading channel model with the Rayleigh fading channel is studied as well. Average capacity keeps growing as the interference temperature increases, and this relationship can be easily seen from Equations (5) and (10) that the selected peak power of SU increases with interference temperature.

In Figures 4 and 5, the number of SUs \( (N_s) \) is chosen as 30. Moreover, it can also be inferred from Figures 4 and 5 that the capacity over the hyper-Nakagami-\( m \) channel achieves higher values than in the Rayleigh fading channel after some certain values of \( P \) and \( Q \) (see Figures 4 and 5). This observation can be justified by Equation (10), which shows that as \( P \) increases, the high power region \( (P \gg Q) \) becomes visible. In this region, as \( P \) increases, the incomplete gamma function part of Equation (10) goes to zero when \( r < P \). In other words, it can be deduced that the SNR at the SUs becomes negatively skewed, and the average power increases. Note that an increase in average power means that there is a decrease in outage probability and finally an increase in capacity. Nevertheless, when \( P < Q \) the PDF of SNR will be different than zero for \( r < P \), and the average power decreases. Note also that decrease in average power means that there is an increase in outage probability and finally a decrease in capacity.
Fig. 6. Average capacity versus fading figure of the channel between the SUs and the PBS $m_\psi$ for different $N_s$ values and fading figure of the channel between the SUs and the SBS $m_\phi$ (1 and 2) when $P = 15\,\text{dB}$ and $Q = 0\,\text{dB}$.

Figures 6 and 7 show the average capacity versus the fading figure of the channel between the SUs and the PBS $m_\psi$ and the fading figure of the channel between the SUs and the SBS $m_\phi$, respectively, with $P = 15\,\text{dB}$, $Q = 0\,\text{dB}$, and Nakagami-$m$ fading channel ($N = 1$). The effect of number of SUs ($N_s$) is also investigated for the values $N_s = 5, 20$ and 40. As $N_s$ increases the capacity increases as well, which can be observed from Equation (15). In Figure 6, with the constant values of $m_\psi$ (i.e., $m_\psi = 1$ and 2) the average capacity decreases as the value of the $m_\phi$ increases, which can be inferred from Equation (10) that the received SNR at the target SBS is reduced. More interestingly, it is observed that there is only slight difference in average capacity while $m_\phi$ increases for the constant the values of $m_\psi$, as shown in Figure 7. The capacity saturates after certain values of $m_\phi$. From this result, it can be concluded that, since the fading figure describes the diversity of the channel, diversity techniques at the SBS do not contribute significantly to the system performance gain. This is one of the important results observed in this work.

Figures 8 and 9 show the average capacity versus the average fading power between the SUs and the SBS ($\gamma_\psi$) and the average fading power between the SUs and the PBS ($\gamma_\phi$) for different $N_s$ values, respectively, for different $N_s$ values with $P = 15\,\text{dB}$, $Q = 0\,\text{dB}$, and hyper-Nakagami-$m$ fading channel with $N = 4$. The same environment accruing factors $\xi$, the fading figure values $m$, and the average fading power between the SUs and the SBS are used as shown in Figure 2. The same
effect of $N_s$ is observed in Figures 8 and 9 as expected, since the capacity increases as $N_s$ increases. In Figure 8, as the $\gamma_\psi$ increases while keeping the $\gamma_\phi$ with two constant values (i.e., $\gamma_\psi = 1$ and 2), the average capacity increases as well. However, for high values of $\gamma_\psi$, this increase reduces. In addition, the effect of $\gamma_\phi$ on average capacity is also investigated in Figure 9 for the two constant values of $\gamma_\psi$ (i.e., $\gamma_\phi = 1$ and 2). It is seen from Figure 9 that the capacity decreases as $\gamma_\psi$ increases for constant values of $\gamma_\phi$. These results can be easily inferred from Equation (10) in the sense that an increase in $\gamma_\psi$ decreases the capacity while an increase in $\gamma_\phi$ increases the capacity.

6. CONCLUSIONS

In this paper, a theoretical fading model that fits to the dynamic nature of spectrum-sharing systems is proposed. The PDF and CDF of the SNR of the SU transmitters at SU receiver along with the PDF of the SU with the highest SNR are derived in closed-forms. The achievable capacity of SU in a spectrum-sharing system is derived for both high and low power regions. The analytical and simulation results are presented to study the effects of fading symmetry and asymmetry in terms of the fading figure and the average power, the number of SUs, and the interference temperature on the capacity of SU in such systems. In spectrum-sharing systems, it is observed that the fading figure of the channel between the SUs and the PBS $m_\psi$ which describes the diversity of the channel does not affect the achievable capacity of the channel significantly. The results show that the proposed model is a promising model that can represent a variety of fading environments in CR systems.

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