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ABSTRACTS

Masanori Adachi
Tokyo University of Science, Japan

Weighted Bergman spaces of Levi-flat domains: two case studies

In contrast to bounded domains in Stein manifolds, it is not clear to what extent Levi-flat bounded domains are capable of holomorphic function with slow growth. We shall answer this question in two cases, the space of geodesic segments and a Grauert tube of a compact Riemann surface, which are realized as Levi-flat bounded 1-convex domains. We describe the weighted Bergman spaces of these domains explicitly.

Florian Bertrand
American University of Beirut, Lebanon

On stationary discs for hypersurfaces in $\mathbb{C}^n$

The study of analytic discs attached to a totally real submanifold M of $\mathbb{C}^n$ leads to the consideration of a Riemann-Hilbert problem. Following this approach, Forstneric, and later on Globevnik, characterized the existence and dimension of a family of deformations of a given analytic disc attached to M in terms of certain indices. However, in case M admits complex points, the indices mentioned above are no longer well-defined and the Forstneric-Globevnik method falls apart. In this talk, I will present a new method based on this geometric approach that allows us to construct stationary discs attached to a large class of hypersurfaces in $\mathbb{C}^n$. This is joint work with Giuseppe Della Sala and Bernhard Lamel.

Debraj Chakrabarti
Central Michigan University, USA

The $\overline{\partial}$-problem in pseudoconcave annuli

We consider the question of solving the $\overline{\partial}$-problem in the annulus bounded by two pseudoconvex domains, where the hole is allowed to be nonsmooth. We obtain estimates for this problem in L^2 spaces using a gluing technique, and prove vanishing results for the L^2? $\overline{\partial}$-cohomology. This is joint work with Mei-Chi Shaw (Notre Dame) and Christine Laurent-Thiébaut (Grenoble).

Giuseppe Della Sala
American University of Beirut, Lebanon

The algebra of germs of solutions of an integrable structure

We consider a locally integrable structure on a smooth manifold M, defined by a suitable subbundle of its complexified tangent bundle, and the ring of germs of solutions of the structure at a point p in M, that is the germs of smooth functions f satisfying $Lf = 0$ for any section L of the subbundle. We study the conditions under which the Borel map, sending a solution to its formal expansion at p, is surjective, and the connection of this property to algebraic aspects such as the locality (and more in general the behavior of the ideals) of the ring of solutions. This is joint work with Paulo Cordaro and Bernhard Lamel.

Makhlouf Derridj
University of Normandy, Rouen, France

On Gevrey vectors of Hörmander's operators

We study the regularity of Gevrey vectors of Hörmander's operators:

$$P = \sum_{j=1}^{m} X_j^2 + X_0 + c,$$

where X_0, X_1, $\ldots$, X_m are real vector fields in an open set $\Omega \subset \mathbb{R}^n$, and c is a smooth function. More precisely, we prove the following: If the coefficients of P are in the Gevrey class $G^k(\Omega)$, $k \in \mathbb{N}$, $k \geq 1$, and P satisfies the following estimate with p/q rational, $0 < p \leq q$:

$$||v||_{p/q}^2 \leq C(|(Pv, v)| + ||v||^2),\ \forall v \in \mathcal{D}(\Omega_0)$$

for some open subset $\Omega_0 \subset \overline{\Omega_0} \subset \Omega$, then $G^k(P, \Omega_0) \subset G^{k\frac{q}{p}}(\Omega_0)$. This provides in particular a local version of a recent result of N. Braun Rodrigues, G Chinni, P. D. Cordaro and M. R. Jahnke, giving a global such result, with $k \geq 1$ not necessarily integer, for Hörmander's operators on a torus.

Son Duong
Texas A&M University at Qatar

The sharp upper bounds for the first positive eigenvalue of Kohn Laplacian on compact strictly pseudoconvex hypersurfaces

In this talk, I will report some results of my joint work with Song-Ying Li and Guijuan Lin regarding the sharp upper bounds for the first positive eigenvalue of Kohn-Laplacian of an embedded strictly pseudoconvex CR manifold. One of the results states that on the family of real ellipsoids, the first positive eigenvalue has a unique maximum at the CR sphere.

Peter Ebenfelt
University of California, San Diego, USA

Umbilical points on strictly pseudoconvex CR manifolds of dimension three

An umbilical point on a strictly pseudonvex CR manifold (of hypersurface type) is a point where the CR structure can be approximated to a higher-than-expected order by that of a sphere (the flat model). In higher dimensions, umbilical points are rare and not stable under perturbations, but in dimension three they are in some sense stable. In this talk, we shall give a family of examples that answer a long-standing question, going back to the fundamental paper of Chern-Moser: Does there exist a compact strictly pseudoconvex hypersurface in $\mathbb{C}^2$ without umbilical points? We will also discuss some results in the opposite direction, providing conditions under which umbilical points on three dimensional compact CR manifolds must exist.

Hanlong Fang
Rutgers University, New Brunswick, USA

Volume preserving holomorphic maps between hermitian symmetric spaces of compact type

We discuss a new rigidity property for local volume preserving maps between hermitian symmetic spaces of compact type along the lines of recent work of Clozel-Ullmo and Mok-Ng. This is a joint work with Prof. X.Huang and Dr. M.Xiao.

Siqi Fu
Rutgers University-Camden, USA

Hearing pseudoconvexity in Lipschitz domains with holes via $\bar{\partial}$

In this talk, we explain how one can determine pseudoconvexity in Lipschitz domains with holes via spectral property of the $\bar{\partial}$-Neumann Laplacian. More precisely, let $\Omega = \widetilde{\Omega} \setminus \overline{D}$ where $\widetilde{\Omega}$ is a bounded domain with connected complement in $\mathbb{C}^n$ and D is relatively compact open subset of $\widetilde{\Omega}$ with connected complement in $\widetilde{\Omega}$. We obtain characterizations of pseudoconvexity of $\widetilde{\Omega}$ and D through the vanishing or Hausdorff property of the Dolbeault cohomology groups on various function spaces. In particular, we show that if the boundaries of $\widetilde{\Omega}$ and D are Lipschitz and C^2-smooth respectively, then both $\widetilde{\Omega}$ and D are pseudoconvex if and only if 0 is not in the spectrum of the $\overline{\partial}$-Neumann Laplacian on $(0,q)$-forms for $1 \leq q \leq n-2$ when $n \geq 3$; or 0 is not a limit point of the spectrum of the $\overline{\partial}$-Neumannn Laplacian on $(0,1)$-forms when $n = 2$. This is a joint work with Christine Laurent-Thiébaut and Mei-Chi Shaw.

Xianghong Gong
University of Wisconsin-Madison, USA

Hölder estimates for homotopy operators on strictly pseudoconvex domains

We will derive a new homotopy formula for strictly pseudoconvex domains in $\mathbb{C}^n$ that admits Hölder estimates. We will also apply the estimates for the homotopy formula to obtain a boundary regularity of the elliptic structure introduced by Treves for the product of real and complex Euclidean spaces.

Friedrich Haslinger
University of Vienna, Austria

The $\overline{\partial}$-Neumann operator, spectral properties and the Rellich lemma

We consider the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is bounded pseudoconvex domain, and

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function. N is the inverse to the complex Laplacian $\Box = \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\,\overline{\partial}$.

Using classical Sobolev inequalities we show that under suitable conditions N can be continuously extended as an operator

$$\tilde{N} : L^{\frac{2n}{n+\epsilon}}_{(0,q)}(\Omega) \longrightarrow L^{\frac{2n}{n-\epsilon}}_{(0,q)}(\Omega),$$

where $0 < \epsilon \leq 1/2$.

In addition, we describe spectral properties of the complex Laplacian $\Box_{\varphi,q}$ on weighted spaces $L^2(\mathbb{C}^n, e^{-\varphi})$. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C} \text{ entire } : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi}\, d\lambda < \infty\}$$

is infinite-dimensional, which depends on the behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ.

We derive a necessary condition for compactness of the corresponding $\overline{\partial}$-Neumann operator and a sufficient condition, both are not sharp. So far, a characterization can only be given in the complex 1-dimensional case.

Furthermore we discuss analogues of the Rellich lemma for $L^2(\mathbb{C}^n, e^{-\varphi})$.

Gustavo Hoepfner
University of Sao Carlos, Brazil

Microlocal regularity of solutions of first-order nonlinear PDEs

In this talk I shall discuss recent advances in the microlocal regularity for solutions of nonlinear PDEs.

Jorge Hounie
University of Sao Carlos, Brazil

Global solvability of real analytic involutive systems on compact manifolds

The focus of this work is the smooth global solvability of a linear partial differential operator $\mathbb{L}$ associated to a real analytic closed non-exact 1-form b – defined on a real analytic closed n-manifold – that may be naturally regarded as the first operator of the complex induced by a locally integrable structure of tube type and co-rank one.

We define an appropriate covering projection $\widetilde{M} \to M$ such that the pullback of b has a primitive $\widetilde{B}$ and prove that the operator is globally solvable if and only if the superlevel and sublevel sets of $\widetilde{B}$ are connected. As a byproduct we obtain a new geometric characterization for the global hypoellipticity of the operator. When M is orientable we prove a connection between the global solvability of $\mathbb{L}$ and that of $\mathbb{L}^{n-1}$ which is the last non-trivial operator of the complex, in particular, we prove that $\mathbb{L}$ is globally solvable if and only if $\mathbb{L}^{n-1}$ is globally solvable. This is joint work with G. Zugliani.

Zhenghui Huo
Washington University in St. Louis, USA

L^p estimate for the Bergman projection on some Reinhardt domains

We start with a star-shaped complete Reinhardt initial domain Ω and generate a class of higher dimensional Reinhardt domains U^α on which the Bergman kernel can be obtained using the kernel function on Ω. We showed that, if certain Schur's test conditions holds for the Bergman kernel on Ω, then a similar condition holds for the Bergman kernel on U^α. We proved, for example, when the domain Ω is a convex domain of finite type in $\mathbb{C}^n$, then the Bergman projection on U^α is bounded in L^p for $1 < p < \infty$. In these cases, the domain U^α may not be of smooth boundary nor be strictly pseudoconvex.

Bernhard Lamel
University of Vienna, Austria

Convergence of formal maps between strictly pseudoconvex CR manifolds

In recent joint work with Nordine Mir, we were able to show that if M and M' are strictly pseudoconvex hypersurfaces in some $\mathbb{C}^N$ and some $\mathbb{C}^{N'}$, respectively, and $H\colon M \to M'$ is a formal map, then H is convergent. We will discuss this result and its proof.

Song-Ying Li
University of California, Irvine, USA

Rigidity theorem for harmonic functions in Bergman metric on bounded symmetric domains

This is a preliminary report on a joint work with R-Y. Chen on the rigidity theorem on the harmonic function in Bergman metric in bounded symmetric domains. A well-known theorem of Graham says that any smooth (C^∞) up to boundary harmonic function in Bergman metric in the unit ball in $\mathbb{C}^n$ must be pluriharmonic. I will present a short survey on the development of such rigidity type theorems for harmonic functions and harmonic map, and, at the same time, I will present recent results with Chen on bounded symmetric domains.

Jeff McNeal
Ohio State University, USA

The Bergman projection on generalized Hartogs triangles

I'll introduce a class of domains that interpolate between the classical Hartogs triangle and the product domain $D \times D*$, and discuss their Bergman theory. The main result is that the Bergman projection, B, of these domains is only bounded on L^p for a restricted range of p. Moreover, that range shrinks to 2 as the domains fill out $D \times D*$. (This is surprising since on $D \times D*$ B maps L^p to itself boundedly for all $1 < p < \infty$.) As time permits, applications to duality theory, holomorphic approximation, and elementary number theory will also be shown. This is joint work with Luke Edholm.

Abdelhamid Meziani
Florida International University, Miami, USA

Planar complex vector fields and deformation of two-dimensional surfaces

We discuss properties of a generalized Pompeiu operator for planar complex vector fields with degeneracies and their application to deformation of two-dimensional surfaces.

Andy Raich
University of Arkansas, USA

The complex Green operator on quadric submanifolds

In this talk, I will survey my recent joint work with Al Boggess. I will discuss the computation of the Box-b heat kernel and how we can compute an integral formula for the complex Green operator. I will also outline our ongoing work the zeroth order asymptotic of the complex Green operator on a particular quadric in $\mathbb{C}^4$ whose estimates do not follow the paradigm of the Heisenberg group.

Michael Reiter
University of Vienna, Austria

A sufficient condition for local rigidity of holomorphic mappings of real submanifolds

Let $M \subset \mathbb{C}^N$ and $M' \subset \mathbb{C}^{N'}$ be germs of real-analytic submanifolds and consider the set $\mathcal{H}$ of germs of holomorphic mappings which locally send M into M'. The group G of local automorphisms of M and M' induces an action on $\mathcal{H}$. A map $H \in \mathcal{H}$ is called locally rigid if all maps sufficiently close to H in $\mathcal{H}$ belong to the G-orbit of H. Assume that $H \in \mathcal{H}$ belongs to the class of finitely nondegenerate mappings. In this case we provide a sufficient linear condition for H which implies local rigidity of H. Our criterion is formulated in terms of the space of holomorphic vectors V such that $\mathrm{Re}\,(V)$ is tangent to M' along the image of H. This is joint work with Giuseppe della Sala and Bernhard Lamel.

Marko Slapar
University of Ljubljana, Slovenia

On the Thom conjecture in $\mathbb{CP}^3$

The Thom Conjecture, proven by Kronheimer and Mrowka in 1994, states that complex curves in $\mathbb{CP}^2$ are genus minimizers in their homology class. In $\mathbb{CP}^n$, an analogous question can be asked about complex hypersurfaces. This question has been studied by Freedman in his thesis, where it was shown that for even $n > 2$, complex hypersurfaces are not the simplest" manifolds in their homology class. We show that in $\mathbb{CP}^3$, complex hypersurfaces of degree greater than 4 do not minimize the second betti number in their homology class. This is joint work with D. Ruberman and S. Strle.

Emil J. Straube
Texas A&M University, College Station, USA

CR submanifolds of hypersurface type: geometry and estimates for $\overline{\partial}_M$

This talk represents an overview of a recent survey concerning the L^2-Sobolev theory of the complex Green operator on a CR submanifold of $\mathbb{C}^n$ of hypersurface type . Some new results concerning propagation of estimates from one bidegree (p, q) to others will also be presented. Both the survey and the new results are joint with S. Biard.

David Tartakoff
University of Illinois, Chicago, USA

On local Gevrey regularity for Gevrey vectors of subelliptic sums of squares

We study the regularity of Gevrey vectors for Hormander operators

$$P = \sum_{j=1}^{m} X_j^2 + X_0 + c$$

where the X_j are real, Gevrey smooth vector fields and $c(x)$ is a smooth function satisfying the subelliptic estimate:

$$\|v\|_\epsilon^2 \leq C|(Pv, v)| + \|v\|_0^2 \qquad \forall v \in C_0^\infty.$$

In 1972, Derridj and Zuily proved G^s hypoellipticity for P for $s > 1/\epsilon = q/p$, and very recently, by studying Gevrey vectors for such operators, Derridj was able to sharpen this result to include $s = q/p$ by showing that for any integer k, the space of Gevrey-k vectors for P, $G^k(P, \Omega) \subset G^{k\frac{q}{p}}(\Omega)$, and to do so, showed (local) Gevrey hypoellipticity for $k \in \mathbb{N}^+$ for $Q = (-1)^k D_t^{2k} - P$ in $G_{t,x}^{1,k\frac{q}{p}}$.

Slightly earlier, N. Braun Rodrigues, G. Chinni, P. D. Cordaro and M. R. Jahnke had obtained a (global) result on a torus for a restricted subclass of P where all coefficients depend only on t.

Here we present a direct proof that for such P, and arbitrary $1 > \epsilon > 0$, and any $s \geq 1/\epsilon$, $G^s(P, \Omega_0) \subset G^{s/\epsilon}(\Omega_0)$. That is,

$$\forall K \Subset \Omega_0,\ \exists C_K : \|P^j u\|_{L^2(K)} \leq C_K^{2j+1}(2j)!^s,\ \forall j$$
$$\implies \forall K' \Subset \Omega_0,\ \exists C_{K'} \|D^m u\|_{L^2(K')} \leq \tilde{C}^{m+1} m!^{s/\epsilon},\ \forall m.$$

The methods also apply to prove the anisotropic result cited above for all k, integral or not.

Note that $G^\ell(\Omega_0) \subset G^\ell(P, \Omega_0)$ always holds.

Ming Xiao
University of Illinois, Urbana Champaign, USA

Embeddability of real hypersurfaces into hyperquadrics and spheres

We discuss the embeddability problems of real hypersurfaces into hyperquadrics and spheres. Among other things, we present a negative answer to a question concerning the embeddability of compact strongly pseudoconvex real algebraic hypersurfaces into spheres, based on joint work with Huang and Li.

Dmitri Zaitsev
Trinity College Dublin, Ireland

A geometric approach to Catlin's boundary systems

We introduce invariant tensors and ideals of order 4 governing Catlin's boundary systems and explain their role in Catlin's proof of subelliptic estimates.

Yunus Zeytuncu
University of Michigan-Dearborn, USA

Compactness of Hankel and Toeplitz operators on domains in $\mathbb{C}^n$

In this talk, I will present various characterizations of compactness of some canonical operators on domains in $\mathbb{C}^n$. I will highlight how complex geometry of the boundary of the domain plays a role in these characterizations. In particular, I will prove that on smooth bounded pseudoconvex Hartogs domains in $\mathbb{C}^2$ compactness of the $\overline{\partial}$-Neumann operator is equivalent to compactness of all Hankel operators with symbols smooth on the closure of the domain. The talk is based on recent joint projects with Željko Čučković and Sönmez Şahutoğlu.