Math 414, Spring 2020
Midterm 1

Name: ____

UIN: ____

1. Calculators are not allowed throughout the examination.
2. Present your solutions in the space provided. Show all your work neatly and concisely.You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it. You have to fully justify every answer.
3. THE AGGIE CODE OF HONOR "An Aggie does not lie, cheat or steal, or tolerate those who do
Student Signature:____________________________________

Question	Value	Score
1	7	
2	7	
3	9	
4	7	
Total	30	

1. (7 points) Let $g(x) = \frac{\pi}{2} - x$ for $0 \leq x \leq \pi$. Find the cosine series of g.

2. (7 points) Consider the inner product space $V = L^2([0,1])$. Let

$$\phi(x) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases} \qquad \psi(x) = \begin{cases} 1, & 0 \le x < 1/2, \\ -1, & 1/2 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) (3 points) Prove that $\{\phi, \psi\}$ is an orthonormal set.

(b) (4 points) Let $f(x) = \sqrt{x}$ for $0 \le x \le 1$. Compute the orthogonal projection of f onto the subspace $V_0 = \text{Span}\,\{\phi, \psi\}$.

3. (9 points) Consider the sequence of functions (f_n) defined on the interval $[0, 1]$ given by

$$f_n(x) = \begin{cases} nx, & 0 \leq x \leq 1/n \\ 1, & 1/n < x \leq 1 \end{cases}$$

(a) (3 points) Let $g(x) = 1$, $0 \leq x \leq 1$. Show that (f_n) converges in $L^2([0, 1])$ to the function g.

(b) (3 points) Show that (f_n) converges pointwise to a function h that you will have to identify.

(c) (3 points) Does the sequence of functions (f_n) converge uniformly to h? No credit will be given for any unjustified answer.

4. (7 points) Prove that if $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are vectors in $\mathbf{C^2}$ then

$$< u, v >= (\bar{v}_1 \ \bar{v}_2) \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

defines an inner product on $\mathbf{C}^2$.