Math 414, Spring 2020
Midterm 2

Name: ____________________

UIN: ____________________

1. Calculators are not allowed throughout the examination.
2. Present your solutions in the space provided. Show all your work neatly and concisely.You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it. You have to fully justify every answer.
3. THE AGGIE CODE OF HONOR "An Aggie does not lie, cheat or steal, or tolerate those who do
   Student Signature:____________________

Question	Value	Score
1	16	
2	4	
3	16	
Total	36	

1. Consider the 2π-periodic extension of the function f given on $[-\pi, \pi]$ by

$$f(x) = \begin{cases} 0, \ -\pi \leq x \leq -\pi/2 \\ x + \dfrac{\pi}{2}, \ -\pi/2 < x \leq 0, \\ \dfrac{\pi}{2} - x, \ 0 < x \leq \pi/2 \\ 0, \ \pi/2 \leq x < \pi \end{cases}$$

We denote by $F(f)$ the Fourier series of f.

(a) (1 point) Sketch the graph of f.

(b) (4 points) State the precise value of $F(f)(x)$ for each x in the interval $-\pi \leq x \leq \pi$. Fully justify your answer.

(c) (3 points) Is the Fourier series $F(f)$ converging uniformly? Explain your answer.

(d) (5 points) Find the Fourier series $F(f)$.

(e) (3 points) Recall what is the general form of Parseval's equation and, using your answer to (d), apply Parseval's equation to the function f under study here to find the value of the following series

$$\sum_{k=1}^{\infty} \frac{(1 - \cos(\frac{k\pi}{2}))^2}{k^4}$$

2. (4 points) Prove the following property about the Fourier transform. Explain all the steps.

   Given a function f differentiable such that $f(t) = 0$ for $|t|$ large:

$$\mathcal{F}[f'](\lambda) = (i\lambda)\mathcal{F}[f](\lambda)$$

3. (16 points) Let

$$\phi(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} \qquad g(x) = \begin{cases} x, & 0 \le x \le 2, \\ 0, & \text{otherwise} \end{cases}$$

(a) (3 points) Compute $\widehat{\phi}(\lambda)$.

(b) (4 points) Compute $\widehat{g}(\lambda)$.

(c) (6 points) Compute $h = \phi * g$.

(d) (3 points) Skech the graph of ϕ, g, h over the interval $[-2, 4]$.