Math 414, Spring 2023
Midterm 2

Name: ____

UIN: ____

1. Calculators, phones, smart-watches any other e-device are not allowed throughout the examination.

2. Present your solutions in the space provided. Show all your work neatly and concisely.You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it. You have to fully justify every answer.

3. Whenever you are using a theorem studied in class, you have to recall the assumptions needed to apply the given theorem and explain very carefully why such assumptions are fulfilled in the situation you are looking at. There are points allocated to providing all such required and detailed explanations.

4. THE AGGIE CODE OF HONOR "An Aggie does not lie, cheat or steal, or tolerate those who do
Student Signature:____________________________________

Question	Value	Score
1	13	
2	12	
3	4	
Clarity/Accuracy	1	
Total	30	

1. (13 points) Let $h(t) = \begin{cases} 1 & 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$ and $g(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$

(a) (2 points) Compute $\widehat{h}$.

(b) (2 points) Compute $\widehat{g}$.

(c) (1 point) Using what you found in parts (a) and (b) and without computing $h * g$, find $\mathcal{F}(h * g)$.

(d) (5 points) Compute $h * g$.

(e) (3 points) Sketch the graphs of h, g and $h * g$.

2. (12 points) Consider the 2π-periodic function defined by $f(x) = x^2$ for $-\pi \leq x < \pi$.

   (a) (2 points) Does the Fourier series of f converge uniformly to f on $[\pi, \pi]$? Explain your reasoning in great details.

   (b) (2 points) Does the Fourier series of f converge pointwise? If yes, what is the pointwise limit? Explain, again, your reasoning in great details.

(c) (4 points) Compute the Fourier series of f.

(d) (2 points) Using what you found in previous questions, show that :

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} = -\frac{\pi^2}{12}$$

(e) (2 points) Using what you found in previous questions, show that:

$$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$$

3. (4 points) State the Riemann-Lebesgue lemma for a continuously differentiable function f on some interval $[a, b]$ and give the proof.