1.2 Graphs of Equations

Graph Analysis

1. Intercepts of a graph:
- Intercepts are the points where the graph touches or intersects the \(x\) — axis or the \(y\) — axis.
- The point where the graph intersects the \(x\) — axis is the \(x\) — intercept and the point where the graph touches the \(y\) — axis is the \(y\) — intercept.
- This means that the \(x\) — intercept is a point where the \(y\) — coordinate is zero. It’s represented as \((a, 0)\), where \(a\) is a point on the \(x\) — axis.
- Similarly, \(y\) — intercept is a point where the \(x\) — coordinate is zero. It’s represented as \((0, b)\), where \(b\) is a point on the \(y\) — axis.

Steps to find the intercepts:

i. For \(x\)-intercept, put \(y=0\) and solve for \(x\).
ii. For \(y\)-intercept, put \(x = 0\) and solve for \(y\).

Ex: Find the \(x\) — and \(y\) — intercepts of the following graphs:

a) \(y = 4x + 3\)
b) \(y = x^2 - 4\)
Side-note: Symmetry of a point: A point \((x, y)\) is said to be symmetrical

- About the x-axis if for the point \((x, y)\) there also exists a point \((x, -y)\)
- About the y-axis if for the point \((x, y)\) there also exists a point \((-x, y)\)
- About the origin if for the point \((x, y)\) there also exists a point \((-x, -y)\)

e.g. Given a point \(A(2, -1)\). Plot its corresponding point such that the point A is
i) Symmetric about \(x\) - axis
ii) Symmetric about \(y\) - axis
iii) Symmetric about the origin
2. Symmetry:
   a) Symmetry about $x$-axis:
      
      **Graphically,** if a graph is folded along the $x$-axis, the portion of the graph above the $x$-axis coincides with the portion below the $x$-axis. That is to say, for every point $(x, y)$ on the graph, there exists a point $(x, -y)$ on the graph, to establish symmetry about $x$-axis.
      
      **Algebraically,** the graph of an equation is symmetric about $x$-axis if replacing $y$ by $-y$ yields an equivalent equation.

   e.g. $x = y^2 + 2$
b) **Symmetry about y-axis:**

**Graphically,** if a graph is folded along the y-axis, the portion of the graph to the left of y-axis coincides with the portion to the right of y-axis. That is to say, for every point \((x, y)\) on the graph, there exists a point \((-x, y)\) on the graph, to establish symmetry about y-axis.

**Algebraically,** the graph of an equation is symmetric about y-axis if replacing \(x\) by \(-x\) yields an equivalent equation.

\[
f(-x) = f(x)
\]

**e.g.** \(y = x^2 - 1\)
b) **Symmetry about the origin:**

**Graphically,** if a graph is folded about the origin, the two portions of the graph on either side of the origin coincide. That is to say, the for every point \((x, y)\) on the graph, there exists a point \((-x, -y)\) on the graph, to establish symmetry about the origin.

**Algebraically,** the graph of an equation is symmetric about origin if replacing \(y\) by \(-y\) and \(x\) by \(-x\) yields an equivalent equation.

\[
f(-x) = -f(x)
\]

**e.g.** \(y = x^3\)
Ques. Test the following for symmetry with respect to $x$—axis, $y$—axis, and the origin

a) $y = \sqrt{x}$

b) $y = x^5 + x^3 + x$

c) $y = |x - 1|$

d) $y = 7x^2 + 12$

e) $y = 3x^3 - 2x$
**Ques:** Use symmetry to sketch the following graphs:
3. **Reflections:**

**Horizontal Reflection (Reflection about the y − axis):** A reflection where the graph flips over HORIZONTALLY. The axis of reflection is Vertical. Every point that was to the right of the y − axis, gets reflected to the left. That is, every $x$ becomes $−x$. Every point that is to the left of the $y −$ axis, gets reflected to the right. That is, every $−x$ becomes $+x$.

**Vertical Reflection (Reflection about the x − axis):** A reflection where the graph flips over VERTICALLY. The axis of reflection is Horizontal. Every point that is above the $x −$ axis, gets reflected to below the $x −$ axis. That is, every $y$ becomes $−y$. Every point that is below the $x −$ axis, gets reflected to above the $x −$ axis. That is, every $−y$ becomes $+y$. 
4. Translations

**Horizontal Translation**: refers to shifting the graph horizontally, to the left or right. This can be done by adding or subtracting a certain number of units from each of the $x$-coordinates of the graph.

**e.g.**

a) $y = |x|

b) $y = |x - 2|

c) $y = |x + 3|$
**Vertical Translation** refers to shifting the graph vertically, upwards or downwards. This can be done by adding or subtracting certain number of units to/from each of the $y$-coordinates of the graph.

a) $y = |x|$

b) $y = |x| - 1$

c) $y = |x| + 3$