Clock offset estimation in wireless sensor networks using robust M-estimation

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ABSTRACT

Clock synchronization plays a crucial role in Wireless Sensor Networks (WSNs). Assuming that there is no clock skew between sensor nodes, the Maximum Likelihood Estimate (MLE) of clock offset was derived by [1] for clock synchronization protocols assuming exponential random delays and a two-way message exchange mechanism as in TPSN (Timing-sync Protocol for Sensor Networks [2]) or NTP (Network Time Protocol). The MLE is appropriate for the case that the random delays in WSNs are exponentially distributed. However, the performance of the MLE is deteriorated considerably in the case that the underlying distribution is contaminated by or mixed with other distributions. Hence, a robust estimator is needed. In this paper, clock offset estimators based on the robust M-estimation method are proposed and it is shown that the proposed estimators present excellent performance in the mean squared error (MSE) sense under the condition that the underlying distribution is mixed with other distribution.

Keywords: Clock Synchronization, Wireless Sensor Networks, Robust M-estimation

1. INTRODUCTION

Wireless sensor networks (WSNs) have received an increased attention due to their promising applications in a variety of areas such as traffic monitoring, surveillance, acoustic and seismic detection, environmental monitoring, etc., and recent advances in micro electro-mechanical systems (MEMS) technology, in digital circuits design, integration and packaging. Wireless sensor networks are composed of a large number of tiny devices, which are networked in an ad-hoc manner without any common infrastructure [3] and are characterized by cheap, unreliable and unattended sensor nodes, limited bandwidth and limited energy resources.

Time synchronization is important for WSNs at many layers of their design. It enables better duty cycling of the radio, accurate localization, beam forming, data fusion, and other collaborative signal processing. The time synchronization problem has been addressed thoroughly for the Internet and local-area-networks (LANs). Many existing algorithms depend on the time information from GPS. However, GPS presents weaknesses: it is not available ubiquitously and requires a relatively high-power receiver, which is almost infeasible on tiny and cheap sensor devices. Besides, the sensor nodes may be left unattended for a considerable period of time, e.g., on the ocean floor or in wild habitats. Furthermore, the limitations of WSNs make it impossible to apply the traditional network synchronization protocols to WSNs. Hence, time synchronization protocols specifically satisfying the constraints of WSNs are needed.

There are generally two stages involved in the time synchronization mechanism of WSNs. The first step is to synchronize the sensor nodes in WSNs to one common absolute time, and the second step is to correct the clock skew relative to a certain standard frequency. The imperfections in the quartz crystal and the environmental conditions make clocks in sensor nodes run at different frequencies, which necessitates the second step. In fact, clock offsets keep drifting away due to the effect of clock skew. Therefore, by correcting each node's clock skew, it is possible to improve the long-term reliability of clock synchronization, and thus reduce energy consumption among the sensor nodes in WSNs. Developing long-term and energy-efficient clock synchronization protocols play a crucial role in deploying successfully long-lasting WSNs. However, in the case that the underlying delay distribution is mixed with or contaminated by other delay distributions, obtaining long-term reliable and energy-efficient clock offset estimators becomes more difficult. In this paper, clock offset estimators based on the robust M-estimation method are proposed. Such estimators are shown to produce better performances in the mean squared error (MSE) for some general mixed delay models.

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2. PROBLEM MODELING

Fig. 1 shows the two-way timing message exchange scheme between two nodes. Node A sends the synchronization message to Node B with its current timestamp $T_{1,i}$ (although it is not required, timestamping in the MAC layer increases the accuracy, as suggested by [2]). Node B stores its current time $T_{2,i}$ at the reception of this message, and then sends at time $T_{3,i}$ a synchronization message to Node A including $T_{2,i}$ and $T_{3,i}$. Node A timestamps the reception time of the message sent by Node B as $T_{4,i}$ (see Fig. 1). Therefore, Node A has access to the set of timestamps $\{T_{1,i}, T_{2,i}, T_{3,i}, T_{4,i}\}$, $i = 1, ..., N$, at the end of $N$ rounds of message exchanges. Note that $T_{1,1}$ is taken as the reference time, and thus every reading $\{T_{1,i}, T_{2,i}, T_{3,i}, T_{4,i}\}$ is actually the difference between the recorded time and $T_{1,1}$.

Several probability distribution functions (pdf) for random queuing delays have been suggested thus far. For example, exponential, gamma, and Weibull distributions were proposed by [7]. [5] experimentally demonstrated the superiority of the MnLD (Minimum Link Delay) algorithm among the various algorithms proposed by [4], which was mathematically proved by [1] assuming exponential delays, thus confirming that the exponential delay assumption fits well the experimental observations. The reason for selecting the Gaussian model is due to the Central Limit Theorem (CLT), which asserts that the pdf of the sum of a large number of independent and identically distributed (iid) random variables (RVs) approaches that of a Gaussian RV. This model is appropriate in the case that the delays are thought to be the addition of numerous independent random processes. The Gaussian distribution for the clock offset errors was also mentioned by a few authors, such as [4], based on laboratory tests.

Next, we assume that there is no clock skew in the two-way timing message exchange model. The $i$th up and down link delay observations corresponding to the $i$th timing message exchange are given by $U_i = T_{2,i} - T_{1,i} = d + \theta_A + X_i$ and $V_i = T_{4,i} - T_{3,i} = d - \theta_A + Y_i$, respectively. The variables $\theta_A$, $d$, $X_i$ and $Y_i$ denote the clock offset between the two nodes, the propagation delay, and variable portions of delays, respectively.

In the exponential delay model and the Gaussian delay model, the MLE of $\theta_A$ exists and is given by

$$\hat{\theta}_A = \frac{\min_{1 \leq i \leq N} U_i - \min_{1 \leq i \leq N} V_i}{2N}$$

and

$$\hat{\theta}_A = \frac{\sum_{i=1}^{N} (U_i - V_i)}{2N},$$

respectively, where $N$ denotes the number of observations of delay measurements [1], [6]. However, these MLEs are not suited for the case that the underlying distribution is mixed with or contaminated by other distributions. Therefore, it is important to estimate the clock offset more accurately in mixed delay models.

3. THE ROBUST M-ESTIMATION METHOD

The procedures of applying robust M-estimation [8] to the clock offset model are as follows. The M-estimator $T_n$ is defined as the solution of the minimization problem, $\min \sum \rho (X_i, \theta)$ where $\rho (\cdot, \cdot)$ is a properly chosen function. If $\rho$ is differentiable with regard to $\theta$ with a continuous derivative $\varphi(\cdot, \theta)$, then $T_n$ is a root (or one of the roots) of the equation,
\[ \sum_{i=1}^{n} \psi(x_i, \theta) = 0, \theta \in \Theta. \] An important special case is the model with the shift parameter \( \mu \), where \( X_1 \ldots X_n \) are independent observations with the same distribution function \( F(x - \theta), \theta \in \Theta \). The distribution function \( F \) is generally unknown. The \( M \)-estimator \( T_\alpha \) is defined as the solution of the minimization, \( \min_{\theta} \sum \rho(X_i - \theta) \) and if \( \rho(\cdot) \) is differentiable with absolutely continuous derivative \( \psi(\cdot) \), then \( T_\alpha \) satisfies the equation, \( \sum \psi(X_i - \theta) = 0 \). If we look for an \( M \)-estimator of the location parameter of a distribution not very far from the normal distribution, but possibly containing an \( \epsilon \) ratio of nonnormal data, more precisely, belonging to the family of distributions, \( \mathbf{F} = \{ F : F = (1- \epsilon)\Phi + \epsilon H \} \), where \( H \) runs over the set of symmetric distribution functions, we should use the function \( \psi_\epsilon(\cdot) \), proposed and motivated by P.J. Huber (1964). This function is linear in the bounded interval \([-k, k]\), and constant outside this interval. \( \psi_\epsilon(x) = x \ldots |x| \leq k \) and \( k \) is a fixed constant, connected with \( \epsilon \) through the following identity: \( 2\Phi(k) - 1 + 2\Phi'(k) / k = 1 / (1- \epsilon) \).

The corresponding \( M \)-estimator is often called the Huber estimator in the literature. It is a robust estimator of the center of symmetry, insensitive to the extreme and outlying observations. As Huber proved in 1964, an estimator, generated by the function \( \psi_\epsilon(\cdot) \), is minimaximally robust for a contaminated normal distribution, while the value \( k \) depends on the contamination ratio. An interesting and natural question is whether there exists a distribution \( F \) such that the Huber \( M \)-estimator is the maximal likelihood estimator of \( \theta \) for \( F(x - \theta) \), i.e., such that \( \psi_\epsilon \) is the likelihood function for \( F \). Such a distribution really exists, and its density is in the interval \([-k, k]\), and exponential outside.

On one hand, if \( X_i \) and \( Y_i \) are i.i.d. exponentially distributed random variables with mean \( \alpha \), \( (U_i - V_i) / 2 \) becomes a zero mean Laplace distributed RV with variance \( \alpha^2 / 2 \). In this case, the \( \psi \) function is \( \psi^*(x) = \text{sign}(x) \ldots |x| \leq k \) and \( 0 \ldots |x| > k \). In the clock offset model, \( (U_i - V_i) / 2 = \theta_A + (X_i - Y_i) / 2 \). This equation can be rewritten as \( (U_i - V_i) / 2 - \theta_A = (X_i - Y_i) / 2 \), where \( U_i = T_{2i} - T_{1i}, \ V_i = T_{4i} - T_{3i} \), \( \theta_A \) denotes the clock offset between two nodes, \( X_i \) and \( Y_i \) denote the variable portions of delays. Therefore, the \( M \)-estimator of the location parameter can be applied to this model.

\[
\sum_{i=1}^{n} \psi\left(\frac{U_i - V_i}{2} - \theta_A\right) = 0.
\]

4. SIMULATION RESULTS

We compared the performances of Huber \( M \)-estimators which use \( \psi^*_\epsilon(\cdot) \) and \( \psi^*(\cdot) \), respectively, under various delay models; i.e., zero-mean Gaussian, exponential, gamma, Weibull, and lognormal delay distributions, respectively, assuming that each distribution is mixed with or contaminated by another zero-mean Gaussian distribution. In this simulation, the rate of contamination or mixing is 20% and the cutoff value of Huber estimation, \( k \) is equal to 1.345. In the following figures, MLE-E and MLE-exponential denote the MLE in the exponential delay model, and MLE-G and CRLB-G denote the MLE and the Cramer-Rao lower bound in the Gaussian delay model, respectively, when no contamination is assumed. In the left figure of Fig. 2, \( \sigma^2_A \) and \( \sigma^2_A \) denote the delay variances for the uplink and the downlink delay distributions, respectively, and \( \sigma^2_B \) and \( \sigma^2_B \) represent the delay variances for the uplink and the downlink Gaussian distributions, which are mixed with the underlying distributions. In the right figure of Fig. 2, and Fig. 3, \( \sigma^2_A \) and \( \sigma^2_A \) indicate the delay variances for the uplink and the downlink Gaussian distributions, which contaminate the underlying distributions.

For the exponential delay model, the mean values \( (\mu_A, \mu_B) \) were chosen to be one. In the gamma delay model, the shape parameter \( (\alpha) \) and the scale parameter \( (\beta) \) were set to be two and one, respectively. For the Weibull case, the shape parameter \( (k) \) and the scale parameter \( (\lambda) \) were set to be two. The exponential, gamma, and Weibull families are considered as representatives to appropriately cover a wide range of typical network delay distributions. Fig. 2 and Fig. 3 consistently show that the clock estimators based on robust M-estimation method present excellent MSE performances in the presence of exponential families of distributions as well as in Gaussian distributions, which are mixed with another Gaussian delay distribution. In other words, the proposed estimators are capable of producing good performances in unknown contaminated delay distributions.
5. CONCLUSIONS

Clock offset estimators based on the robust M-estimation method are proposed in a clock synchronization protocol involving a two-way message exchange model. It is clearly illustrated that the robust M-estimation based clock offset estimators present good MSE performance under several delay distributions that assume contaminated distributions.

Fig. 2. MSEs of clock offset estimators for the Gaussian and exponential delay models with contaminated Gaussian delays

Fig. 3. MSEs of clock offset estimators for the gamma and Weibull delay models with contaminated Gaussian delays

REFERENCES


