Diversity combining with up-link power control

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Summary

We introduce in this paper a new adaptive power-controlled diversity combining scheme that reduces the average transmitted power of the mobile units (MUs) while meeting a certain minimum required quality of service. The key idea is (i) to collect and combine all the available diversity paths at the base station (BS) and then (ii) to request the MU to increase or decrease its transmitted power just to track the required target signal-to-noise ratio (SNR). Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are proposed and studied. Some selected numerical results show that the proposed scheme offers considerable savings in the transmitted power levels over a wide SNR range but amplifier saturation leads to a violation of the target BER requirement in the low average SNR range. Additional numerical examples show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: fading channel; diversity techniques; selection combining; power control

1. Introduction

Wireless communication systems are subject to a harsh propagation environment which leads to frequent fading dips in the received signal. These tough conditions make reliable communication very hard, and, as a result, various fading countermeasure techniques are needed to improve the performance of these systems. For instance, power control and diversity combining are typically used in existing and emerging wireless communication systems to mitigate the problem of signal power fading. In this paper, inspired by the mode of operation of power control algorithms in the 3GPP standard [1], we propose and study an up-link power-controlled diversity combining (UPC-DC) scheme. As its name indicates, UPC-DC combines the features of classical diversity combining with some up-link power control from the mobile unit (MU) to the base station (BS). UPC-DC capitalizes first on diversity combining by collecting and combining all the available diversity paths at the BS and then (ii) to request the MU to increase or decrease its transmitted power just to track the required target signal-to-noise ratio (SNR). Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are proposed and studied. Some selected numerical results show that the proposed scheme offers considerable savings in the transmitted power levels over a wide SNR range but amplifier saturation leads to a violation of the target BER requirement in the low average SNR range. Additional numerical examples show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm. Copyright © 2007 John Wiley & Sons, Ltd.

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‡ This is an extended version of work which was accepted for presentation in 2006 IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC’06), Helsinki, Finland, September 2006.

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paths at the BS. Subsequently and based on the resulting combined signal-to-noise ratio (SNR), the BS requests via a feedback path the MU to increase or decrease its transmitted power just to track a particular required target SNR. In this paper, we study the performance of the proposed scheme and show how this scheme reduces the average bit error rate (BER) with, of course, an attendant (and quantifiable) small increase in transmitted power only in the low average SNR range.

The remainder of the paper is organized as follows. Section 2 presents the system and channel models then gives the details behind the mode of operation of various variants of UPC-DC. While Section 3 provides some analytical results, Section 4 illustrates these results via some selected figures. Finally, Section 5 offers some concluding remarks.

2. Models and Mode of Operation

2.1. System Model

We consider a generic diversity system with \( L \) available diversity paths. This includes for example, RAKE receivers which are used in wideband CDMA systems to combine the available resolvable multipaths. For hardware complexity considerations, we assume that up to \( L_c \) branches can be combined at the receiver side (i.e., the number of fingers of the RAKE receiver is limited to \( L_c \)). We also assume that the proposed UPC-DC scheme has a reliable feedback path between the receiver and the transmitter and is implemented in a discrete-time fashion. More specifically, and as shown in Figure 1, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the receiver performs a series of operation, including (i) path estimation, (ii) combined SNR comparison with respect to the predetermined SNR threshold, and (iii) request to the MU power amplifier to increase or decrease its gain by a specific amount. Once all the available diversity paths are selected and once the appropriate transmitted power is reached, the combiner (at the receiver end) and the power amplifier (at the transmitter) are configured accordingly and this transmitter and receiver settings are used throughout the subsequent data burst.

2.2. Channel Model

We denote by \( \gamma_l \) \((l = 1, 2, \ldots, L)\), the received SNR of the \( l \)th diversity path (under nominal transmitted power from the MU) and, as illustrated in Figure 1, we adopt a block flat fading channel model. More specifically, assuming slowly varying fading conditions, the different diversity paths experience roughly the same fading conditions (or equivalently the same SNR) during the data burst and its preceding guard period. In addition, the fading conditions are assumed to (i) be independent across the diversity paths and between different guard period and data burst pairs and (ii) follow anyone of the popular fading models such as Rayleigh, Rice, or Nakagami-\( m \).

2.3. Mode of Operation of UPC-DC

If the number \( L \) of available paths in the BS is below the number \( L_c \) of paths that can be combined, the BS combines all the available paths as per the rules of maximal-ratio combining (MRC) [8]. On the other hand, if the number \( L \) of available diversity paths exceeds the number \( L_c \) of paths that can be combined, the BS uses generalized selection combining (GSC) (see for example References [2–5]). With GSC, the \( L_c \) (among the \( L \) available) diversity branches with the best quality (quantified for example in terms of fading amplitude or equivalently instantaneous SNR) are selected and combined in an MRC fashion. At the beginning of the guard period, the MU power amplifier gain \( G \) (with respect to the nominal transmitted power) is initially set to 0 dB and based on this setting the combining process described above is performed. If the combiner fails to meet the \( \gamma_T \) requirement during the combining process phase, the receiver activates the power control mechanism and requests the transmitter to increase its gain in order to meet the target SNR requirement. If on the other hand, the required output SNR \( \gamma_T \) is reached during this initial phase, then the receiver activates also the power control mechanism.

\[ \text{Fig. 1. Block fading channel model.} \]

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\[ \text{DOI: 10.1002/wcm} \]
but requests in this case the transmitter to decrease its gain such that the output combined SNR just matches the target SNR requirement. We consider in our study, four power adaptation variants:

1. **Continuous adaptation without amplifier gain saturation**: In this first ideal case, we assume that the gain of the MU amplifier $G$ can be adjusted in a continuous fashion and is not limited by any maximal value.

2. **Continuous adaptation with amplifier gain saturation**: In this case, we still assume that the gain of the transmitter amplifier can be adjusted continuously but saturates to a certain maximal value $G_{\text{max}}$. In the case, that a gain beyond $G_{\text{max}}$ is needed to meet the required target SNR, we assume that the MU units deactivates the power control mechanism and transmits with the nominal power level. This is done to save some valuable battery lifetime but comes of course at the expense of the violation of the target SNR requirement in very adverse channel fading conditions.

3. **Discrete adaptation without amplifier gain saturation**: Similar to the power control algorithms that are implemented in the 3 GPP standard, we assume in this case that the MU amplifier gain can only take discrete values. This gain can be adjusted using a binary feedback and a power control step size $G_{\text{step}}$.

4. **Discrete adaptation with amplifier gain saturation**: In this most practical case, we assume that the gain takes discrete values and saturates to a fixed maximal value $G_{\text{max}}$. We again assume that the MU transmits with nominal power level if a gain beyond $G_{\text{max}}$ is needed to meet the required target SNR.

### 3. Performance Analysis

#### 3.1. Statistics of the Combined SNR

Regardless of the type of adaptation used, the probability density function (PDF), $p_{\gamma_{\text{out}}} (\cdot)$, of the combined SNR at the end of the diversity combining stage is given by

$$p_{\gamma_{\text{out}}} (\gamma) = (1 - P_L(L_c))p_{\gamma_{\text{max}}} (\gamma) + P_L(L_c)p_{\gamma_{\text{gsc}}} (\gamma)$$

where $P_L (l) = P[L \leq l]$ is the cumulative distribution function (CDF) of the number of available diversity paths in the area of deployment, which can be, for example, modeled by a Poisson distribution [6,7], $p_{\gamma_{\text{max}}} (\cdot)$ is the PDF at the output of an $L$-branch MRC diversity combiner which is known in closed-form for many fading scenarios of interest [8], and $p_{\gamma_{\text{gsc}}} (\cdot)$ is the PDF at the output of a GSC receiver combining the $L_c$ strongest branches among the $L$ available ones which is also known in closed-form for many fading scenarios of interest [8].

With the PDF of the combined SNR available, we can find the average BER and the additional average dB gain $G_{\text{dB}}$ that is required by the up-link power control for the four power adaptation variants under consideration. In the following we provide the final analytical formulas as well as some selected numerical results illustrating the performance of our proposed UPC-DC. Detailed derivations of the formulas are given in Appendices.

#### 3.2. Continuous Adaptation Without Amplifier Gain Saturation

In this case, the average BER is constant and equal to $\text{BER}(\gamma_T)$, where $\text{BER}(\gamma)$ is the BER of the modulation when used over an additive white Gaussian noise (AWGN) channel with SNR $\gamma$. The corresponding additional average dB gain can be shown to be given by

$$G_{\text{dB}} = \gamma_{T\text{dB}} - 10\int_{0}^{\infty} \log_{10}(\gamma)p_{\gamma}(\gamma) \, d\gamma$$

For independent identically distributed (i.i.d.) Rayleigh fading conditions and $L_c \geq L$ (i.e., the receiver combines the $L$ available diversity paths in an MRC fashion), the combined SNR is given by [10, Equation (6.23)] as

$$p_{\gamma_c}(\gamma) = \frac{1}{(L - 1)! \bar{\gamma}^L} \gamma^{L - 1} e^{-\frac{\gamma}{\bar{\gamma}}}$$

where $\bar{\gamma}$ is the average SNR per symbol.

Inserting Equation (3) in (2), it can be shown with the help of [9, Equation (4.352.1)] and [9, Equation (8.365.4)] that

$$G_{\text{dB}} = \gamma_{T\text{dB}} - \frac{10}{\ln 10} \left( -C + \sum_{l=1}^{L-1} \frac{1}{l} + \ln \bar{\gamma} \right)$$

where $C = 0.577215664$ is the Euler constant.  

\footnote{The additional average dB gain $G_{\text{dB}}$ is defined as the average of the additional dB gain and is therefore given by $G_{\text{dB}} = E[\gamma_{T\text{dB}} - \gamma_{\text{dB}}]$.}
3.3. Continuous Adaptation with Amplifier Gain Saturation

In this case, the average BER can be shown to be given by

\[ \text{BER} = \int_{0}^{\gamma_T/G_{\text{max}}} \text{BER}(\gamma)p_{\gamma_c}(\gamma) \, d\gamma + \text{BER}(\gamma_T)(1 - P_{\gamma_c}(\gamma_T/G_{\text{max}})) \]  

(5)

where \( P_{\gamma_c}(\cdot) \) is the CDF of the combined SNR at the end of the combining phase and which is known to be given by [10, Equation (6.25)] as

\[ P_{\gamma_c}(\gamma) = \int_{0}^{\gamma} p_{\gamma_c}(x) \, dx = 1 - \frac{\Gamma(L, \frac{\gamma}{G_{\text{max}}})}{(L-1)!} \]  

(6)

where \( \Gamma(\cdot, \cdot) \) is the incomplete gamma function defined for positive integer \( n \) in [9, Equation (8.352.2)] as

\[ \Gamma(n, x) = (n-1)! \sum_{m=0}^{n-1} \frac{x^m}{m!} \]  

(7)

and for general real \( \alpha \) as [9, Equation (8.350.2)]

\[ \Gamma(\alpha, x) = \int_{x}^{+\infty} t^{\alpha-1} e^{-t} \, dt \]  

(8)

For binary phase shift keying (BPSK), the BER(\( \gamma \)) over AWGN is known to be given by the following

\[ \text{BER} (\gamma) = 0.5 \, \text{erfc} \left( \sqrt{\gamma} \right) \]  

(9)

where \( \text{erfc}(\cdot) \) is the complementary error function defined in [9, Equation (8.350.2)] as

\[ \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{+\infty} e^{-t^2} \, dt \]  

(10)

Inserting Equations (3), (6), and (9) in (5), (5) can be re-written in closed-form by using [9, Equation (2.321.2)] followed by integration by part yielding

\[ \text{BER} = 0.5 \left[ -\text{erfc}(\sqrt{\gamma}/G_{\text{max}}) \frac{\Gamma(L, \frac{\gamma_T}{G_{\text{max}}})}{(L-1)!} \right]_{0}^{\gamma_T/G_{\text{max}}} + 0.5 \sum_{l=0}^{L-1} \frac{1}{\sqrt{\pi} l!} \left( \frac{1}{2} \gamma_T \mu^{-l+\frac{1}{2}} \right) \left[ \Gamma \left( l + \frac{1}{2}, \mu \gamma_T/G_{\text{max}} \right) \right]_{0}^{\gamma_T/G_{\text{max}}} \]

(11)

where \( \mu = 1 + \left( \frac{1}{2} \right) \).

The corresponding additional average dB gain can be shown to be given by

\[ G_{\text{dB}} = \gamma_T \left( 1 - P_{\gamma_c}(\gamma_T/G_{\text{max}}) \right) - 10 \int_{0}^{\gamma_T/G_{\text{max}}} \text{log}_{10}(\gamma)p_{\gamma_c}(\gamma) \, d\gamma \]  

(12)

For i.i.d. Rayleigh fading conditions and \( L_c \geq L \), we show in Appendix 2 that Equation (12) can be written in the desired closed form as

\[ G_{\text{dB}} = \frac{\gamma_T^{L_c}}{(L-1)!} \Gamma(L, \frac{\gamma_T}{G_{\text{max}}}) \]

\[ - 10 \ln 10 \ln \frac{L}{(L-1)!} \sum_{l=0}^{L-1} \frac{\Gamma(l, \frac{\gamma_T}{G_{\text{max}}})}{l!} \]  

(13)

3.4. Discrete Adaptation Without Amplifier Gain Saturation

In this case, the average BER can be shown to be given by

\[ \text{BER} = \sum_{k=-\infty}^{+\infty} \int_{0}^{10^{-k\gamma_{\text{dB}}/10}} \frac{\gamma_{\text{dB}}(k-1)G_{\text{dB}}}{10} \, p_{\gamma_c}(\gamma) \, d\gamma \]

(14)
For i.i.d. Rayleigh fading conditions and $L_c \geq L$, we show in Appendix 3 that
\[
\text{BER} = \sum_{k=\infty}^{+\infty} 0.5 \left[ -\text{erfc}(\sqrt{a(k)\gamma}) \frac{\Gamma(L, \frac{\gamma}{\beta})}{(L-1)!} \right] + \sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{\pi} \left( \frac{1}{\gamma} \right)^l \times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right)\]
\[
\times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right) \right]\]  

where $a(k) = 10^{\frac{kG_{db}}{10}}$ and $\beta = a(k) + \left( \frac{1}{\gamma} \right)$.

The corresponding additional average dB gain can be easily shown to be given by
\[
G_{db} = \sum_{k=\infty}^{+\infty} P_{\gamma_c} \left( 10^{\frac{\gamma_T - (k-1)G_{db}}{10}} \right) G_{\delta_{db}} \]
(16)

For i.i.d. Rayleigh fading conditions and $L_c \geq L$, we get after using Equation (6) the desired closed-form result as
\[
G_{db} = \sum_{k=\infty}^{+\infty} \left( -\frac{1}{\gamma} \right)^l \left\{ \frac{\Gamma(L, \frac{\gamma}{\beta})}{(L-1)!} \right\} G_{\delta_{db}} \]
(17)

### 3.5. Discrete Adaptation with Amplifier Gain Saturation

In this case, the average BER can be shown to be given by
\[
\text{BER} = \sum_{k=\infty}^{K_M + 1} \left[ -\text{erfc}(\sqrt{a(k)\gamma}) \frac{\Gamma(L, \frac{\gamma}{\beta})}{(L-1)!} \right] + \sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{\pi} \left( \frac{1}{\gamma} \right)^l \times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right)\]
\[
\times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right) \right]\]  

where $K_M = \frac{G_{\max_{db}}}{G_{\delta_{db}}}$. For i.i.d. Rayleigh fading conditions and $L_c \geq L$, we can get the desired closed form by applying Equations (11) and (15) to (18)
\[
\text{BER} = \sum_{k=\infty}^{K_M} 0.5 \left[ -\text{erfc}(\sqrt{a(k)\gamma}) \frac{\Gamma(L, \frac{\gamma}{\beta})}{(L-1)!} \right] \]
\[
+ \sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{\pi} \left( \frac{1}{\gamma} \right)^l \times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right)\]
\[
\times \beta^{-l+\frac{1}{2}} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right) \right]\]  

where $\mu = 1 + \frac{1}{\gamma}$
\[
\beta = a(k) + \frac{1}{\gamma}
\]
\[
a(k) = 10^{\frac{kG_{db}}{10}}
\]
\[
K_M = \frac{G_{\max_{db}}}{G_{\delta_{db}}} \]

The corresponding additional average dB gain can be easily shown to be given by
\[
G_{db} = \sum_{k=\infty}^{K_M + 1} f(k)P_{\gamma_c} \left( 10^{\frac{\gamma_T - (k-1)G_{db}}{10}} \right) G_{\delta_{db}} \]
(20)

where
\[
f(k) = \begin{cases} 
1 & k = -\infty, \ldots, K_M \\
-K_M k & K_M + 1 
\end{cases} \]
(21)

For i.i.d. Rayleigh fading conditions and $L_c \geq L$, inserting Equation (6) in (20), it can be shown

DOI: 10.1002/wcm
that

\[ G_{\text{dB}}(\gamma) = \frac{1}{\Gamma(L, 10^{\frac{\gamma T_{\text{dB}}}{\gamma}})} \left( \sum_{k=-\infty}^{K_M+1} f(k) \left( 1 - \frac{10^{\frac{\gamma T_{\text{dB}}}{\gamma}}(k-1) G_{\delta \text{dB}}}{(L-1)!} \right) G_{\delta \text{dB}} \right) \]

(22)

4. Numerical Examples

Figure 2 compares the BER of BPSK when used with MRC and no power control and when used in conjunction with continuous UPC-DC with and without amplifier gain saturation. Clearly UPC makes the system just meet the target BER over the whole SNR range while systems without PC either fail to meet this target BER in the low average SNR region or exceeds it in the high average SNR region. In addition, when UPC is used the saturation of the transmitter amplifier leads to a violation of the target BER requirement in the low average SNR region. However, we can see from Figure 3 that this peak power constraint at the transmitter side leads to a considerable decrease in the required additional average transmitter gain in this same low average SNR region.

While Figures 4 and 5 compare continuous and discrete power adaptation (with different step sizes) without amplifier gain saturation, Figures 6 and 7 do the same comparison when there exists a peak power constraint at the transmitter side. One can see from these figures, that as long as \( G_{\text{max \text{dB}}} \) is an integer multiple of \( G_{\delta \text{dB}} \) discrete power control requires a slightly higher average gain but offers correspondingly a decrease in the average BER over the whole average SNR range. However if the value of \( G_{\text{max \text{dB}}} \) is not an integer multiple of \( G_{\delta \text{dB}} \), this behavior is reversed in the low average SNR range.

Fig. 2. Average BER of BPSK with (i) continuous UPC-DC and no amplifier gain saturation, (ii) continuous UPC-DC and amplifier gain saturation, and (iii) MRC and no power control (\( L_c = L = 6, \gamma_T = 5 \text{ dB}, \) and \( G_{\text{max}} = 3 \text{ dB} \)).
Fig. 3. Average additional dB gain of the transmitter amplifier using continuous UPC-DC with and without saturation ($L_c = L = 6$, $\gamma_T = 5$ dB, and $G_{\text{max}} = 3$ dB).

Fig. 4. Average BER of BPSK with (i) continuous UPC-DC and no amplifier gain saturation and (ii) discrete UPC-DC and no amplifier gain saturation ($L_c = L = 6$ and $\gamma_T = 5$ dB).
Fig. 5. Average additional dB gain of the transmitter amplifier with (i) continuous UPC-DC and no amplifier gain saturation and (ii) discrete UPC-DC and no amplifier gain saturation ($L_c = L = 6$ and $\gamma_T = 5$ dB).

Fig. 6. Average BER of BPSK with (i) continuous UPC-DC and amplifier gain saturation and (ii) discrete UPC-DC and amplifier gain saturation ($L_c = L = 6$, $\gamma_T = 5$ dB, and $G_{max} = 4$ dB).
5. Conclusion

In this paper, we proposed a new adaptive up-link diversity combining scheme. The key idea is to take advantage of all the diversity offered by the channel and then request the transmitter (i) to increase its power level during very adverse channel conditions in order to reach the target SNR and (ii) to decrease its power level during favorable channel conditions just to keep the SNR level at the target required SNR. Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation were proposed and studied. For each power control variant, we obtained a closed-form solution for the average BER and the required additional average dB gain. Based on some selected results, we showed that our proposed scheme makes the system meet the target BER over the whole SNR range but the amplifier saturation leads to a violation of the target BER requirement in the low average SNR range. However, these results also show that our proposed scheme offer considerable savings in the transmitted power levels over a wide SNR range even when the practical implementation constraints such as amplifier saturation and/or discrete power adaptation are taken into account.

Acknowledgment

This work was supported by Qatar Telecom (Qtel), Qatar.

Appendix 1: A Useful Identity

In what follows, we give a useful formula that we will use in our derivations. Starting from [9, Equation (2.231.2)], we can write

\[
\int \gamma^{n-\frac{1}{2}} e^{-\gamma} \, d\gamma = -e^{-\gamma} \left( \frac{\gamma^{n-\frac{1}{2}}}{\mu} + \sum_{k=1}^{n-\frac{1}{2}} \frac{1}{\mu^{k+1}} \right)
\times \frac{(n - \frac{1}{2})!}{(n - (k + \frac{1}{2}))!} \gamma^{n-(k+\frac{1}{2})}
\]

\[
= -e^{-\gamma} \sum_{k=0}^{n-\frac{1}{2}} \frac{1}{\mu^{k+1}} \frac{(n - \frac{1}{2})!}{(n - (k + \frac{1}{2}))!} \gamma^{n-(k+\frac{1}{2})}
\]

\[
(23)
\]
Let \( n - \left( k + \frac{1}{2} \right) = \alpha \), then Equation (23) can be re-written as

\[
\int \gamma^{n-\frac{1}{2}} e^{-\gamma} \, d\gamma = -\frac{\mu}{\mu + \frac{1}{\alpha}} \sum_{\alpha=0}^{n-\frac{1}{2}} \frac{1}{\alpha!} (n-\frac{1}{2})! \gamma^\alpha
\]

(24)

Let \( \alpha = k \), then Equation (24) can be shown with the help of [9, Equation (3.381.3)] and [9, Equation (3.381.3)] to be equal to

\[
\int \gamma^{n-\frac{1}{2}} e^{-\gamma} \, d\gamma = -\mu^{-(n+\frac{1}{2})} \Gamma \left( n + \frac{1}{2}, \mu \gamma \right)
\]

(25)

**Appendix 2: Derivation of Equation (13)**

In this Appendix, we derive a closed-form expression for the average BER in the case of continuous adaptation with amplifier gain saturation mode of UPC-DC. This gain is given in this case by Equation (14). Using Equations (6) and (3) in (12), we can write

\[
G_{dB} = \frac{\gamma T_{dB}}{(L-1)!} \Gamma \left( L, \frac{\gamma T}{G_{max}} \right) + \frac{1}{(L-1)!} \gamma L \int_{\gamma T_{max}}^{\infty} \gamma^{L-1} \ln (\gamma) e^{-\gamma} \, d\gamma
\]

(26)

Let \( \frac{\gamma T}{G_{max}} = x \), then Equation (26) can be re-written as

\[
G_{dB} = \frac{1}{(L-1)!} \gamma T \int_{1}^{\infty} \left( \frac{\gamma T}{G_{max}} \right)^{L} x^{L-1} \ln \left( \frac{\gamma T}{G_{max}} + \ln x \right) e^{-\frac{\gamma T}{G_{max}} x} \, dx
\]

\[
= \frac{1}{(L-1)!} \left( \frac{\gamma T}{G_{max}} \right)^{L} \ln \left( \frac{\gamma T}{G_{max}} \right) \int_{1}^{\infty} x^{L-1} e^{-\frac{\gamma T}{G_{max}} x} \, dx
\]

\[
+ \frac{1}{(L-1)!} \left( \frac{\gamma T}{G_{max}} \right)^{L} \int_{1}^{\infty} x^{L-1} \ln (x) e^{-\frac{\gamma T}{G_{max}} x} \, dx
\]

(27)

Finally, using [9, Equation (2.321.2)] and [9, Equation (3.381.3)], we get the desired closed-form result given in Equation (13).

**Appendix 3: Derivation of Equation (15)**

In this Appendix, we derive a closed-form expression for the average BER in the case of discrete adaptation without amplifier gain saturation mode of UPC-DC. The average BER is given in this case by Equation (14). Let \( a(k) = 10^{G_{dB}/10} \), then

\[
\int \text{erfc} \left( \sqrt{a(k)} \gamma \right) p_{\gamma} (\gamma) \, d\gamma
\]

\[
= \text{erfc} \left( \sqrt{a(k)} \gamma \right) - \sum_{l=0}^{L-1} \left( \frac{\gamma}{\gamma} \right)^{l} \frac{1}{l!} e^{-\frac{\gamma}{\gamma}}
\]

(28)

\[
- \int \frac{d}{d\gamma} \left( \text{erfc} \left( \sqrt{a(k)} \gamma \right) \right) \left( -\sum_{l=0}^{L-1} \left( \frac{\gamma}{\gamma} \right)^{l} \frac{1}{l!} e^{-\frac{\gamma}{\gamma}} \right) \, d\gamma
\]

(29)

Using Equation (7)

\[
\sum_{l=0}^{L-1} \left( \frac{\gamma}{\gamma} \right)^{l} \frac{1}{l!} e^{-\frac{\gamma}{\gamma}} = \frac{\Gamma \left( L, \frac{\gamma}{\gamma} \right)}{(L-1)!}
\]

(30)

and

\[
\frac{d}{d\gamma} \text{erfc} \left( \sqrt{a(k)} \gamma \right) = -\sqrt{a(k)} \gamma^{-\frac{1}{2}} e^{-a(k)\gamma},
\]

(31)

Equation (29) can be re-written as

\[
-\int \frac{d}{d\gamma} \left( \text{erfc} \left( \sqrt{a(k)} \gamma \right) \right) \left( -\sum_{l=0}^{L-1} \left( \frac{\gamma}{\gamma} \right)^{l} \frac{1}{l!} e^{-\frac{\gamma}{\gamma}} \right) \, d\gamma
\]

\[
= -\int -\sqrt{a(k)} \gamma^{-\frac{1}{2}} e^{-a(k)\gamma} \left( -\sum_{l=0}^{L-1} \left( \frac{\gamma}{\gamma} \right)^{l} \frac{1}{l!} e^{-\frac{\gamma}{\gamma}} \right) \, d\gamma
\]

\[
= -\sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{l!} \left( \frac{1}{\gamma} \right)^{l} \int \gamma^{\frac{1}{2}} e^{-\left( a(k) + \frac{1}{2} \right) \gamma} \, d\gamma
\]

(32)

Using Equation (25) in (32), we can write

\[
-\sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{l!} \left( \frac{1}{\gamma} \right)^{l} \int \gamma^{\frac{1}{2}} e^{-\left( a(k) + \frac{1}{2} \right) \gamma} \, d\gamma
\]

\[
= -\sum_{l=0}^{L-1} \sqrt{a(k)} \frac{1}{l!} \left( \frac{1}{\gamma} \right)^{l} \beta^{-\left(l+\frac{1}{2} \right)} \Gamma \left( l + \frac{1}{2}, \beta \gamma \right)
\]

(33)

where \( \beta = a(k) + \left( \frac{1}{\gamma} \right) \).
Combining the results of Equations (14), (28), (29), (30), and (33), we get the desired closed-form result given in (15).

References


Authors’ Biographies

Sung Sik Nam was born in Korea. He received his B.S. and M.S. degrees in Electronic Engineering from Hanyang University, Korea in 1998 and 2000, respectively. Also he received his M.S. degree in Electrical Engineering from University of Southern California (USC), Los Angeles, CA, U.S.A. in 2003. Since August 2005, he has been pursuing the Ph.D. degree at Texas A&M University (TAMU), College Station, TX, U.S.A. From 1998 to 1999, he worked as a Researcher at The Electronics & Telecommunication Research Institute (ETRI), Daejeon, Korea. From 2003 through 2004, he worked as Manager at the Korea Telecom Corporation (KT), New Business Planning Office, New Business Planning Team, Korea. On November 2006, he joined Texas A&M University at Qatar (TAMUQ), Doha, Qatar, where he is working toward the Ph.D. degree under Qatar Telecom (QTEL) projects. His research interests include the design and performance analysis of wireless communications, diversity techniques, power control, multiuser scheduling.

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Khalid A. Qaraqe was born in Bethlehem. Dr Qaraqe received his B.S. degree in EE from the University of Technology, Baghdad in 1986, with honors, he received the M.S. degree in EE from the University of Jordan, Jordan in 1989, and he earned his Ph.D. degree in EE from Texas A&M University, College Station, TX, in 1997. From 1989 to 2004 Dr Qaraqe has held a variety positions in many companies and he has over 12 years of experience in the telecommunication industry. He has worked for Qualcomm, Enad Design Systems, Cadence Design Systems/Tality Corporation, STC, SBC, and Ericsson. He has worked on numerous GSM, CDMA, WCDMA projects and he has experience in product development, design, deployments, testing, and integration. Dr Qaraqe joined the department of Electrical Engineering of Texas A&M University at Qatar in July 2004, where he is now a visiting Associate Professor. His research interests include performance analysis of the 3G UMTS wireless communication, WCDMA estimation theories, fading channels, frequency hopping, and STTD diversity.