Joint Adaptive Modulation and Diversity Combining with Feedback Error Compensation

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Abstract—This paper investigates the effect of feedback error on the performance of the joint adaptive modulation and diversity combining (AMDC) scheme which was previously studied with an assumption of error-free feedback channels. We also propose to utilize adaptive diversity to compensate for the performance degradation due to feedback error. We accurately quantify the performance of the joint AMDC scheme in the presence of feedback error, in terms of the average number of combined paths, the average spectral efficiency, and the average bit error rate. Selected numerical examples are presented and discussed to illustrate the effectiveness of the proposed feedback error compensation strategy with adaptive combining. It is observed that the proposed compensation strategy can offer considerable error performance improvement with little loss in processing power and spectral efficiency in comparison with the no compensation case.

Keywords—Adaptive modulation, diversity techniques, feedback error, and performance analysis.

I. INTRODUCTION

Because of the growing demand for a higher spectral efficiency as well as a good link reliability in wireless communications, many new technologies have been proposed over the last few decades. Among them, adaptive transmission aims to optimize the transmission rate according to the fading channel variations. It has been shown that a considerable gain in throughput can be achieved by using adaptive transmission while maintaining a certain target error rate performance and assuming an error-free feedback channel [1]–[3]. On the other hand, diversity technique refers to a method for improving the reliability of wireless fading channels by utilizing two or more communication channels with different characteristics and as such, it plays an important role in combating fading and co-channel interference. Various classical diversity combining techniques can be found in [4]–[6].

Recently, as an attempt to obtain further improved spectral efficiency under the same error rate requirement, some joint adaptive modulation and diversity combining (AMDC) schemes were proposed and analyzed in [7], [8]. Unlike [9], [10] which are some earlier AMDC works focusing on the channel capacity, [7], [8] employ more sophisticated diversity combining techniques such as generalized selection combining (GSC) [11], minimum selection-GSC (MS-GSC) [12], and output threshold-maximum ratio combining (OT-MRC) [13]. With the schemes in [7], [8], the receiver jointly decides the proper modulation mode and diversity combiner structure based on the channel quality and the target error rate requirement. Different mode of operations have been considered based on the primary optimization criteria of the joint design. For example, the power-efficient AMDC scheme leads to a high processing power efficiency, the bandwidth-efficient AMDC scheme leads to a high bandwidth efficiency, and the bandwidth-efficient and power-greedy AMDC scheme leads to a high bandwidth efficiency as well as an improved power efficiency at the cost of a higher error rate in comparison with the bandwidth-efficient AMDC scheme.

In most of the published papers dealing with adaptive modulation, it is generally assumed for analytical tractability that the feedback channel is error-free. However, there are a number of real-life scenarios in which this ideal assumption is not valid, especially in the case that sufficient and powerful error control method cannot be implemented over the feedback channel. As such, the study of the impact of imperfect feedback channels is very important. For instance, in [14], [15], the impact of imperfect feedback channels on the performance was investigated and two feedback-detection strategies have been proposed to mitigate the performance degradation and to reduce the outage region due to feedback errors. In this paper, we take a different point of view from that of [14], [15] and show that receiver diversity can be utilized to compensate for feedback error with the adjustment of the combiner structure.

We first consider the effect of feedback error on the performance of the joint AMDC systems and study the usage of adaptive combining at the receiver to mitigate this effect. We assume that feedback error may induce that the adaptive modulation mode used for transmission may be different from the one selected by the receiver after adaptive combining. To accommodate for such scenarios, the transmitter needs to inform the receiver which mode it will be using before the actual data transmission. We propose to use diversity path adjustment at the receiver to mitigate the error performance degradation or explore additional power savings. Specifically,

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the receiver may combine more diversity paths, if possible, to compensate for the bit error rate (BER) degradation when the transmitter sends data with a higher modulation mode than the one selected by the receiver. On the other hand, the receiver may use less diversity paths to save the receiver processing power when the transmitter sends data with a lower modulation mode. Hence, the number of combined paths can be adaptively changed depending on the nature of feedback error. We investigate the impact of this compensation method by analyzing the average number of combined paths, the average spectral efficiency, and the average BER of the joint AMDC scheme proposed in [7] in the presence of feedback error with and without adaptive combining path adjustment. We show through selected numerical examples that the proposed compensation strategy can offer considerable error performance improvement with little loss in processing power and spectral efficiency in comparison with the no compensation case.

II. CHANNEL AND SYSTEM MODEL

A. Signal and Channel Model

Basically, we adopt the same signal and channel model as in [7]. We assume that the joint AMDC scheme is implemented in a discrete-time fashion. More specifically, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the receiver performs a series of operations, including path estimations and combined signal-to-noise ratio (SNR) comparisons with respect to the predetermined SNR threshold. After determining the most appropriate diversity combiner structure and adaptive modulation mode to be used during the subsequent data burst, the receiver sends back the adaptive modulation mode to the transmitter via reverse link before the guard period ends. Because of feedback channel imperfection, the modulation mode used for transmission may be different from the one selected by the receiver. We assume that the transmitter informs the receiver about the actual transmitting mode before transmission.

We also assume slow frequency flat fading channels and adopt a block fading model where the fading coefficients are assumed to be constant through the data burst period. As such, all the diversity paths experience almost the same fading conditions and maintain therefore the same SNR during the data burst and its preceding guard period. Moreover, the fading conditions are assumed to follow the Rayleigh model and to be independent and identically distributed (i.i.d.) across the diversity paths and between different guard periods and data bursts. Hence, if we let \( \gamma_i \) denote the instantaneous received SNR of the \( i \)th path, \( i = 1, 2, \cdots, L \), where \( L \) is the number of available diversity paths at the receiver, then the faded SNR, \( \gamma_i \), follows the same exponential distribution with the common average faded SNR, \( \gamma \) [5, Eq. (6.5)].

B. Joint Adaptive Modulation and Diversity Combining

We adopt the constant-power variable-rate uncoded \( M \)-ary quadrature amplitude modulation (\( M \)-QAM) scheme\(^1\), where the modulation mode, \( M \), is restricted to a power of 2, \( 2^n \).

Assume that the SNR range is divided into \( N + 1 \) regions and each region is associated with a particular QAM signal constellation. The region boundaries, denoted by \( \gamma_{T_n} \), are set to the SNR required to achieve the target BER, denoted by \( \text{BER}_0 \), using \( 2^n \)-QAM over an additive white Gaussian noise (AWGN) channel. It has been shown that the instantaneous BER of \( 2^n \)-QAM with two-dimensional Gray coding over an AWGN channel with SNR of \( \gamma \) can be well approximated by [2, Eq. (28)]

\[
\text{BER}_n(\gamma) = \frac{1}{5} \exp \left( -\frac{3\gamma}{2(2^n - 1)} \right), \quad n = 1, 2, \cdots, N. \quad (1)
\]

Therefore, the boundary thresholds can be calculated in terms of a target BER, \( \text{BER}_0 \), as

\[
\gamma_{T_n} = \begin{cases} \frac{2}{3} \ln(5\text{BER}_0)(2^n - 1), & n = 0, 1, \cdots, N; \\ \infty, & n = N + 1. \end{cases} \quad (2)
\]

The receiver chooses the most suitable modulation mode, \( n \), or constellation size, \( M = 2^n \), based solely on the fading channel conditions. This is done by first examining the received SNR and then finding a proper region in which its estimated SNR falls. If the estimated SNR is in the \( n \)th region, the receiver informs via feedback path the corresponding modulation mode to the transmitter so that the constellation size of \( 2^n \) is used during the subsequent data burst.

Now we consider the joint AMDC schemes proposed in [7]. Specifically, we focus on the bandwidth-efficient and power-greedy AMDC scheme [7, Sec. V] since it has been shown to provide the highest spectral efficiency among the three schemes and better power efficiency than the bandwidth-efficient AMDC scheme. The basic principle behind the bandwidth-efficient and power-greedy AMDC scheme is to combine the smallest number of diversity paths such that the highest achievable modulation mode can be used while satisfying the instantaneous BER requirement.

More specifically, during the guard period, the receiver estimates all \( L \) diversity paths and ranks them yielding the ordered SNRs, \( \gamma_{1:L} > \gamma_{2:L} > \cdots > \gamma_{L:L} \), where \( \gamma_{k:j} \) is the \( j \)th order statistics among \( j \) ones (see [11] for terminology). By using the MS-GSC scheme [12], the receiver first finds the highest modulation mode that can be used based on the current fading channel condition. Once the modulation mode is selected, then the receiver turns off as many of the weakest paths as possible such that the combined SNR of the remaining paths is large enough for the selected modulation mode to be used. If, in the worst case, the combined SNR of all \( L \) available paths is still below \( \gamma_{T_1} \), the receiver may ask the transmitter to either transmit using the lowest modulation mode in violation of the target instantaneous BER requirement (option 1), or buffer the data and wait until the next guard period for more favorable channel conditions (option 2).

In what follows, because of space limitations, we consider only option 2 and omit specific derivations. Analysis and detailed derivations for both options can be found in the journal version [16].

\(^1\)Using the adaptively coded schemes in [3], our results can be easily extended to coded systems.
III. PATH ADJUSTMENT FOR FEEDBACK ERROR COMPENSATION

In this section, a detailed algorithm of the feedback error compensation for the bandwidth-efficient and power-greedy AMDC scheme is proposed. Assume that after performing a series of operations described in the previous section, the receiver decides to use the modulation mode $j$ ($0 \leq j \leq N$) with $l$ ($0 \leq l \leq L$) strongest paths. The receiver sends the index of this modulation mode, $j$, to the transmitter via the feedback channel but due to a possible feedback error the transmitter may end up using mode $n$ ($0 \leq n \leq N$) instead of $j$ for transmission. We assume that the transmitter always informs the receiver about the actual transmitting mode, i.e., the receiver has the knowledge of $n$ before the transmission occurs. We propose to use path adjustment for the case of feedback error. If $j > n$, for $n > 0$ the receiver reduces the combined paths by sequentially removing the weakest paths until either the combined SNR satisfies the new output threshold, $\gamma_{T_n}$, or the number of combined paths is 1. Note that for option 2 with $n = 0$, no transmission occurs. If $j < n$, then the receiver will combine all $L$ paths in order to maximally improve the system performance since based on the mode of operation the AMDC scheme, the output SNR with $L$ combined paths should still be between $\gamma_{T_j}$ and $\gamma_{T_{j+1}}$ (i.e., $\gamma_{T_j} \leq \Gamma_{L:L} \leq \gamma_{T_{j+1}}$ where $\Gamma_{i:j}$ is the sum of the $i$ largest SNRs among $j$ ones, i.e., $\Gamma_{i:j} = \sum_{k=1}^{i} \gamma{k:j}$). Finally, no path adjustment is necessary if $j = n$. The path adjustment algorithm for the bandwidth-efficient and power-greedy AMDC scheme with option 2 in the presence of feedback error is depicted in Fig. 1.

IV. PERFORMANCE ANALYSIS IN THE PRESENCE OF FEEDBACK ERROR

In this section, we analyze the performance of the bandwidth-efficient and power-greedy AMDC scheme with feedback error over Rayleigh fading channels. More specifically, we obtain its average number of combined paths, average spectral efficiency, and average BER. In the following analysis, we denote $q_{n,j}$ as the mode transition probability which is the probability of using mode $n$ instead of $j$ due to feedback error.

A. Average Number of Combined Paths

As mentioned in Section III, if $j > n$, the receiver will reduce the combined paths while all $L$ paths are combined when $j < n$. Let $P_{l,n,j}$ denote the probability that mode $n$ is used with $l$ combined paths while the true constellation size is $j$. Then, the average number of combined paths with feedback error is given for option 2 by

$$N_c = \sum_{l=1}^{L} l \sum_{j=1}^{N} q_{n,j} P_{l,n,j}$$

$$+ L \sum_{j=0}^{N-1} \sum_{n=j+1}^{N} q_{n,j} \Pr[\gamma_{T_j} < \Gamma_{L:L} < \gamma_{T_{j+1}}].$$

(3)

Specific details for the derivation of $P_{l,n,j}$ can be found in [16].

B. Average Spectral Efficiency

In case of no feedback error, the average spectral efficiency, $\eta$, of the adaptive system can be calculated as [2, Eq. (33)]

$$\eta = \sum_{n=1}^{N} n \pi_n,$$  

(4)

where $\pi_n$ is the probability that $n$th modulation mode is used which can be expressed for the bandwidth-efficient and power-greedy AMDC scheme as [7]

$$\pi_n = F_{MSC}(\gamma_{T_n})(\gamma_{T_{n+1}}) - F_{MSC}(\gamma_{T_n})(\gamma_{T_n}),$$  

(5)

where $F_{MSC}(\gamma_{T_n})(\cdot)$ denotes the cumulative distribution function (CDF) of the combined SNR with $L$-branch MS-GSC and using $\gamma_{T_n}$ as an output threshold and which is given for i.i.d Rayleigh fading channels in [12, Eq. (20)]. With feedback error, we can obtain the average spectral efficiency as

$$\eta = \sum_{n=1}^{N} \sum_{j=0}^{N-1} n q_{n,j} \pi_j,$$  

(6)

where $q_{n,j}$ is the mode transition probability due to feedback error and $\pi_j$ is given above in (5).
C. Average Error Rate

The average BER for the adaptive modulation system can be calculated as [2, Eq. (35)]

\[
\text{BER} = \frac{1}{\eta} \sum_{n=1}^{N} n \cdot \text{BER}_n, \tag{7}
\]

where \(\text{BER}_n\) is the average error rate when constellation size \(n\) is used for transmission. When there is feedback error and the system does not adjust the number of combined paths, \(\text{BER}_n\) can be calculated as

\[
\text{BER}_n = \sum_{j=0}^{n} q_{n,j} \int_{\gamma_{T_j}}^{\gamma_{T_{j+1}}} \text{BER}_n(x)f_{\gamma}(x)dx, \tag{8}
\]

where \(f_{\gamma}(x)\) is the probability density function (PDF) of the output SNR which has been obtained in [7, Eq. (23)]. If the system adjusts the combiner structure for the case of feedback error, specific care needs to be done to calculate \(\text{BER}_n\), since in this case the PDF of the output SNR should be changed. Specifically, we consider the case of \(j < n\) and \(j \geq n\) separately. Let \(f_{\gamma}^-(x)\) denote the PDF of the output SNR after adjustment for \(j < n\) case and \(f_{\gamma}^+(x)\) for \(j \geq n\) case. Then, \(\text{BER}_n\) for option 2 can be written as

\[
\text{BER}_n = \sum_{j=0}^{n-1} q_{n,j} \int_{\gamma_{T_j}}^{\gamma_{T_{j+1}}} \text{BER}_n(x)f_{\gamma}^-(x)dx \tag{9}
\]

\[
+ \sum_{j=n}^{N} q_{n,j} \int_{\gamma_{T_n}}^{\gamma_{T_{n+1}}} \text{BER}_n(x)f_{\gamma}^+(x)dx. \tag{9}
\]

Noting that the receiver combines all \(L\) paths when the actual mode \(n\) is greater than the feedback mode \(j\), the PDF, \(f_{\gamma}^-(x)\), is simply the PDF of the \(L/L\)-GSC output SNR [6, Eq. (9.433)]. On the other hand, when \(j \geq n\), the receiver will combine less paths as long as the output SNR is greater than \(\gamma_{T_n}\). The PDF of the output SNR, \(f_{\gamma}^+(x)\), over the range of \([\gamma_{T_n}, \gamma_{T_{n+1}}]\) can be found in [16]. Finally, substituting (9) into (7) leads to the desired average BER of the bandwidth-efficient and power-greedy AMDC scheme for option 2 with feedback error over Rayleigh fading channels.

V. NUMERICAL EXAMPLES

In this section, we illustrate the analytical results derived in the previous section with some selected numerical examples. In what follows, we use the transition probability, \(q_{n,j}\), obtained from the PSK-based sequential labeling scheme presented in [16, Sec. III-B and Table 1] where \(q_{n,j}\) is calculated in terms of the average BER, \(P_b\), of the feedback channel. Note that all numerical evaluations obtained from the analytical results derived in this paper have been compared by Monte Carlo simulations of the system under consideration (for the better visual quality, we plot the simulation results only for \(P_b = 10^{-1}\) in all figures).

In Fig. 2, we present the average number of combined paths of the bandwidth-efficient and power-greedy AMDC scheme with option 2 for different average BERs, \(P_b\), of the feedback channel as a function of the average SNR per path, \(\gamma\), when \(L = 5, N = 3\), and \(\text{BER}_0 = 10^{-3}\).

In Fig. 3, we present the average spectral efficiency of the bandwidth-efficient and power-greedy AMDC scheme with option 2 for different average BERs, \(P_b\), of the feedback channel as a function of the average SNR per path, \(\gamma\), when \(L = 5, N = 3\), and \(\text{BER}_0 = 10^{-3}\). From this figure, we can observe that if the average BER of the feedback channel is relatively small, i.e., \(P_b = 10^{-2}\) and \(10^{-3}\), the average number of combined paths is almost the same as that in the case of no feedback error, which means in this case feedback error does not significantly affect the performance measure. On the other hand, when \(P_b = 10^{-1}\), we can see that the system requires more paths over the low SNR region and less paths over the high SNR region. Note that as the average BER
VI. CONCLUSIONS

In this paper, we studied the joint AMDC scheme for high spectral and power efficient wireless systems, taking into account imperfect feedback channel conditions. More specifically, it is assumed that the modulation mode the transmitter receives from the receiver can be perturbed by some feedback errors that can result in the usage of the wrong transmission mode between the transmitter and the receiver. In order to mitigate the performance impediment, the receiver adjusts its diversity combining structure according to the nature of feedback error. Through the analysis of this scheme, we have shown that our proposed path adjustment method can reduce the probability of error degradation while maintaining almost the same processing power and spectral efficiency in comparison with the scheme without path adjustment.

REFERENCES