Minimum Selection GSC with Down-Link Power Control

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Abstract—We propose and analyze in this paper two new adaptive power controlled minimum selection generalized selection combining (MS-GSC) diversity schemes. The key idea behind these two schemes is to request the transmitter to vary its power level during very adverse channel conditions in order to communicate with the minimum required quality of service. For each scheme, four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are proposed and studied. Selected numerical examples, show that ideal continuous power control lowers the average bit error rate of MS-GSC and makes it always meet the specified target quality of service at the expense of an increase in the transmitter gain. Additional numerical examples, show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm.

Index Terms—Diversity techniques, minimum selection generalized selection combining, down-link power control, Rayleigh fading channels, performance analysis.

I. INTRODUCTION

DIVERSITY combining is a classical concept which has been used for the past half century to combat the effects of fading on wireless systems. Over the last decade, low-complexity diversity combining schemes operating in a diversity rich environment received a great deal of attention. Among these schemes, generalized selection combining (GSC), also known as hybrid selection/maximum ratio combining (H-S/MRC), was the first to be proposed (e.g. [1]–[5]). With GSC, the $L_c$ (among the $L$ available) diversity branches with the best quality (quantified for example in terms of fading amplitude or equivalently signal-to-noise ratio (SNR)) are selected and combined as per the rules of maximal-ratio combining (MRC). Subsequently and as a power-saving implementation of GSC in the power/size limited mobile units (MUs), minimum selection GSC (MS-GSC) was proposed in [6] and recently further studied and analyzed in [7]–[9]. With MS-GSC the receiver ranks the SNR of all available paths and then combines the minimum number of branches (up to $L_c$) in order to make the combined SNR exceed a certain predetermined threshold. While the MS-GSC receiver is still constituted of $L_c$ branches (like GSC), these branches do not need to be always active, and as such MS-GSC can save (in an average sense) a considerable amount of processing power and increase the valuable battery time of the MUs. However, in very adverse channel conditions, it can happen that MS-GSC fails to guarantee the minimum required quality of service in the case the combined SNR of the $L_c$ strongest diversity branches does not exceed the predetermined SNR threshold.

Inspired by the mode of operation of downlink power control algorithms in the 3GPP standard, we propose and study an extension of MS-GSC (termed power controlled MS-GSC (PC-MS-GSC)). In our study, we consider two schemes of PC-MS-GSC, namely (i) post-combining power control scheme (PCPC) and (ii) joint combining and power control scheme (JCPC). In these two schemes we combine the features of MS-GSC as proposed for MUs in [6] with some downlink power control from the base station (BS) to the MU. In this paper, we analyze the performance of these newly proposed schemes by relying on a new result on the joint distribution of two consecutive cumulative sums of ordered statistics, we then illustrate these performance results via some selected numerical examples.

The remainder of the paper is organized as follows. Section II presents some assumptions defining the system and the channel model, gives the mode of operation of each proposed diversity scheme, and defines different adaptation variants used by the transmitter during the power control phase. While section III provides some analytical results, section IV offers some selected numerical examples. Finally, section V offers some concluding remarks.
II. MODELS AND MODE OF OPERATION

A. Channel and System Model

We assume that there are $L$ diversity paths available at the receiver. The signal on each diversity path experiences independent identically distributed (i.i.d.) Rayleigh fading. As such, the faded SNR, denoted by $\gamma_i$ ($i = 1, 2, \ldots, L$), follows an exponential distribution, with common probability density function (PDF) and cumulative distribution function (CDF) given by

$$f_{\gamma_i}(x) = \frac{1}{\bar{\gamma}} \exp \left( -\frac{x}{\bar{\gamma}} \right), \quad x \geq 0$$

and

$$F_{\gamma_i}(x) = 1 - \exp \left( -\frac{x}{\bar{\gamma}} \right), \quad x \geq 0,$$

respectively, where $\bar{\gamma}$ is the common average faded SNR.

We adopt a block flat fading channel model. More specifically, assuming slowly-varying fading conditions, the different diversity paths experience roughly the same fading conditions (or equivalently the same SNR) during a data burst and its preceding guard period.

Because of hardware constraints, we assume that the receiver can only afford an $L_{c}$-branch MRC combiner, where ($L_{c} \leq L$). The receiver chooses a proper subset of diversity paths from the available ones and process them in a MRC fashion. We assume that the proposed PC-MS-GSC schemes have a reliable feedback path between the receiver and the transmitter and are implemented in a discrete-time fashion. More specifically during the short guard period before each data burst, the receiver estimates the SNR of all available diversity paths and determines which paths to be MRC-combined during data burst reception.

B. Mode of Operation

1) PCPC: At the beginning of the guard period, the transmitter HPA gain $G$ (with respect to the nominal transmitted power) is initially set to 0 dB and based on this setting the receiver starts by estimating, ranking, then combining the diversity paths in a MS-GSC fashion. If the required output SNR $\gamma_T$ is reached during this initial phase, then no additional high power amplifier (HPA) gain is needed and the receiver is configured during the subsequent data burst time with the suitably selected diversity branches. If, on the other hand, the MS-GSC combiner fails to meet the $\gamma_T$ requirement during this initial phase, the receiver activates the power control mechanism and requests the HPA to increase its gain in a similar fashion as it is done with PCPC.

2) JCPC: In this case, if the threshold is reached during the initial phase the receiver will ask the transmitter to reduce its power by the amount $\gamma_c - \gamma_T$, and then the transmitter will provide only the power level allowing to communicate with the minimum required quality of service. If, on the other hand,

3) Power Adaptation Variants: We now introduce the four power adaptation variants under consideration.

   a) Continuous Adaptation without Saturation: In this most ideal case, we assume that the gain of the HPA $G$ can be adjusted in a continuous fashion and is not limited by any maximal value.

   b) Continuous Adaptation with Saturation: In this case, we still assume that the gain of the transmitter HPA can be adjusted continuously but saturates to a certain maximal value $G_{\text{max}}$.

   c) Discrete Adaptation without Saturation: Similar to the power control algorithms that are implemented in the 3 GPP standard, we assume in this case that the HPA gain can only take discrete values. This gain can be adjusted using a binary feedback and a power control step size $G_{\text{step}}$.

   d) Discrete Adaptation with Saturation: In this most practical case, we assume that the gain takes discrete values and saturates to a fixed maximal value $G_{\text{max}}$.

For continuous and discrete adaptations with saturation we have two options depending on the behavior of the HPA.

- Option 1: In this option, if the receiver asks the transmitter for more than a maximum value of $G$ ($G_{\text{max}}$) the power control is not activated. It means that for this first option there is no down-link power control during extreme fading conditions for which the required output SNR $\gamma_T$ can not be met even if the maximum amplifier gain is used.

- Option 2: In this option, if the receiver asks the transmitter for more than a maximum value of $G$ ($G_{\text{max}}$) the power control is activated. It means that for this second option there is no down-link power control during extreme fading conditions for which the required output SNR $\gamma_T$ can not be met even if the maximum amplifier gain is used.

\[ \text{Start} \]

\[ \gamma = 0 \quad k = 0 \]

\[ \text{Estimate/Rank} \]

\[ \gamma_i, \quad i = 1, \ldots, L \]

\[ k = k + 1 \]

\[ \text{Update MRC output SNR} \]

\[ \gamma_c = \gamma_c + \gamma k : L \]

\[ \text{HPA} \uparrow \text{your Gain by } \gamma_T - \gamma_c \]

\[ \text{Yes} \]

\[ k < L_c \]

\[ \text{Yes} \]

\[ \text{Stop} \]

\[ \text{No} \]
III. PERFORMANCE RESULTS

We provide in what follows some performance results for the various variants of our proposed PCPC and JPC schemes which were double checked for their accuracy via Monte-Carlo simulations.

A. Post-Combining Power Control

1) Continuous Adaptation without Saturation: In this case, the average BER can be shown to be given by

$$\text{BER} = \int_{\gamma_T}^{\infty} \text{BER}(\gamma) f_{\gamma_e}(\gamma) d\gamma + \text{BER}(\gamma_T) F_{\gamma_e}(\gamma_T),$$

where $f_{\gamma_e}(\cdot)$ and $F_{\gamma_e}(\cdot)$ are the CDF and the PDF of the combined SNR with GSC, respectively. The PDF and the CDF of GSC are given, for the case of Rayleigh fading, in [4, Eqs. (24), (27)] by

$$f_{\gamma_e}(\gamma) = \sum_{l=0}^{L-1} \frac{a_l}{\gamma^{l+1}} e^{-\gamma/l}, \quad \gamma \geq 0,$$

$$F_{\gamma_e}(\gamma) = 1 - \sum_{l=0}^{L-1} A_l \frac{\gamma^l}{l!} e^{-\gamma/l}, \quad \gamma \geq 0,$$

respectively, where

$$a_l = \left( \begin{array}{c} L \cr L_c \end{array} \right) \sum_{k=0}^{L_c} \left( \begin{array}{c} L - L_c \cr k \end{array} \right) \frac{(-1)^k L_c - 1}{k^{l+1}},$$

$$b_k = \left( \begin{array}{c} L \cr L_c \end{array} \right) \frac{(-1)^k L_c - 1}{k^{l+1}},$$

$$A_l = \sum_{i=0}^{l} a_{l+i} = A_l + A_{l+1}, \text{ with } A_{-1} = 0,$$

$$B_k = \frac{L_c b_k}{L_c + L}.$$

It can be easily shown that the corresponding additional average dB gain can be given by

$$\overline{G}_{dB_1} = \ln 10 \int_{0}^{\gamma_T/G_{\max}} x 10^{\frac{\gamma_T - \gamma}{10}} f_{\gamma_e} \left( 10^{\frac{\gamma_T - \gamma}{10}} \right) dx.$$  

(10)

A closed-form expression for $\overline{G}_{dB_1}$ can be obtained by a change of variable and with the help of [10, Eqs. (3.351.2), (4.331.2)] and [11, Eq. (64), Appendix A], and is given by

$$\overline{G}_{dB_1} = \sum_{l=0}^{L-1} \frac{a_l}{k!} \ln 10 \left[ \frac{L_c e^{-\gamma/l}}{k^{l+1}} \right] \left( 1 - \gamma_T e^{-\gamma/l} \right)$$

$$- \frac{10}{\ln 10} \left[ \Gamma \left( k, \frac{\gamma_T}{\gamma} \right) - \Gamma \left( k, \frac{\gamma_T}{\gamma} \right) - \ln \left( \frac{\gamma_T}{\gamma} \right) \right]$$

$$- \sum_{k=1}^{L-L_c} \frac{b_k}{1 + \frac{k}{L_c}} \left[ \frac{\gamma_{th} e^{\gamma_T/k}}{1 + \frac{k}{L_c}} - e^{\gamma_T/k} \right]$$

$$- \frac{10}{\ln 10} \left[ E_i \left( -\frac{1 + k}{\gamma_T} \right) - E_i \left( -\frac{1 + k}{\gamma_T} \right) \right]$$

$$- \ln \left( \frac{\gamma_T}{\gamma} \right) \right].$$  

(11)

where $\Gamma(\cdot, \cdot)$ is the complementary incomplete Gamma function and $E_i(\cdot)$ is the exponential integral function [10].

2) Continuous Adaptation with Saturation: For this mode of adaptation we have two options:

a) Option 1: If the combined SNR is such that $\gamma_T/G_{\max} \leq \gamma_e \leq \gamma_T$ then power control is activated and the combined SNR is set to $\gamma_T$. For this option, the average BER is given by

$$\overline{\text{BER}_1} = \int_{0}^{\gamma_T/G_{\max}} \text{BER}(\gamma) f_{\gamma_e}(\gamma) d\gamma$$

$$+ \text{BER}(\gamma_T) \left[ F_{\gamma_e}(\gamma_T) - F_{\gamma_e}(\gamma_T/G_{\max}) \right]$$

$$+ \int_{\gamma_T}^{\infty} \text{BER}(\gamma) f_{\gamma_e}(\gamma) d\gamma.$$  

(12)

The corresponding additional average dB gain can be easily shown to be given by

$$\overline{G}_{dB_1} = \ln 10 \int_{0}^{\gamma_T/G_{\max}} x 10^{\frac{\gamma_T - \gamma}{10}} f_{\gamma_e} \left( 10^{\frac{\gamma_T - \gamma}{10}} \right) dx.$$  

(13)

Using a change of variable, [10, Eqs. (3.351.2), (4.331.2)], and [11, Eq. (64), Appendix A] a closed-form expression for $\overline{G}_{dB_1}$ with option 1 can be shown to be given by

$$\overline{G}_{dB_1} = \sum_{l=0}^{L-1} \frac{a_l}{k!} \left[ G_{\max, dB_1} \left( \frac{\gamma_T}{G_{\max}} \right) e^{-\frac{\gamma_T}{G_{\max}}} \right]$$

$$- \frac{10}{\ln 10} \left[ \Gamma \left( k, \frac{\gamma_T}{G_{\max}} \right) - \Gamma \left( k, \frac{\gamma_T}{\gamma} \right) - \ln \left( \frac{\gamma_T}{\gamma} \right) \right]$$

$$- \sum_{k=1}^{L-L_c} \frac{b_k}{1 + \frac{k}{L_c}} \left[ \frac{\gamma_{th} e^{\gamma_T/k}}{1 + \frac{k}{L_c}} - e^{\gamma_T/k} \right]$$

$$- \frac{10}{\ln 10} \left[ E_i \left( -\frac{1 + k}{\gamma_T} \right) - E_i \left( -\frac{1 + k}{\gamma_T} \right) \right]$$

$$- \ln \left( \frac{\gamma_T}{\gamma} \right) \right].$$  

(14)
b) Option 2: If $\gamma_c \leq \gamma_T/G_{\text{max}}$ power control is activated with a maximum HPA gain and the linear combined SNR is set to $\gamma_c$, $G_{\text{max}}$, and the average BER is given in this case by

$$\overline{\text{BER}}_2 = \int_0^{\gamma_T} \text{BER}(\gamma) \frac{1}{G_{\text{max}}} f_{\gamma_c}(\gamma/G_{\text{max}}) d\gamma$$

$$+ \text{BER}(\gamma_T) \left( F_{\gamma_c}(\gamma_T) - F_{\gamma_c}(\gamma_T/G_{\text{max}}) \right)$$

$$+ \int_{\gamma_T}^{\infty} \text{BER}(\gamma) f_{\gamma_c}(\gamma) d\gamma. \quad (15)$$

The corresponding additional average dB gain can be easily deduced from the result of option 1 as

$$\overline{G_{\text{dB}}} = \overline{G_{\text{dB}}} + G_{\text{max}} \text{Pr}[\gamma_c < \gamma_T/G_{\text{max}}]$$

$$= \overline{G_{\text{dB}}} + G_{\text{max}} F_{\gamma_c}(\gamma_T/G_{\text{max}})$$

$$= \ln\left(\frac{10}{10} \int_0^{G_{\text{max}} G_{\text{dB}}} x 10^{(\gamma_T - \gamma x)/10} f_{\gamma_c}(10^x/10) dx \right)$$

$$+ G_{\text{max}} G_{\text{dB}} F_{\gamma_c}(\gamma_T/G_{\text{max}}). \quad (16)$$

Similar to the previous option, a closed-form of $\overline{G_{\text{dB}}}$ can also be obtained in this case and can be deduced from (14) and is given by

$$\overline{G_{\text{dB}}} = \sum_{l=0}^{L_c-1} \sum_{k=0}^l a_l G_{\text{max}}^k e^{-\gamma_T/G_{\text{max}}} \left(1 - G_{\text{max}} e^{-\gamma_T/G_{\text{max}}} \right)$$

$$- \frac{10}{\ln(10)} \left( \Gamma(k, \gamma_T/G_{\text{max}}) - \Gamma(k, \gamma_T) \right)$$

$$- \ln(G_{\text{max}}) e^{-\gamma_T/G_{\text{max}}}$$

$$- \sum_{k=1}^{L_c - L_c} B_k \left[ G_{\text{max}} e^{-1 + \gamma_T} - e^{-1 + \gamma_T} \right]$$

$$- \frac{10}{\ln(10)} \left[ Ei\left(1 - \frac{k}{\gamma_T} \right) - Ei\left(1 + \frac{k}{\gamma_T} \right) \right]$$

$$+ \frac{G_{\text{max}} G_{\text{dB}}}{G_{\text{max}}}. \quad (17)$$

3) Discrete Adaptation without Saturation: We define $G_\delta$ as the power control step. Using the mode of operation of PCPC, it can easily be shown that the average BER is given by

$$\overline{\text{BER}} = \int_{\gamma_T}^{\infty} \text{BER}(\gamma) f_{\gamma_c}(\gamma) d\gamma$$

$$+ \sum_{k=0}^{\infty} \int_{\gamma_T}^{10^{G_{\text{dB}} - k G_{\text{dB}}/10}} \text{BER}\left(10^{\gamma_{\text{dB}}/10}\right) f_{\gamma_c}(\gamma) d\gamma, \quad (18)$$

and that the corresponding additional average dB gain is given by

$$\overline{G_{\text{dB}}} = \sum_{k=0}^{\infty} k \frac{G_{\text{dB}}}{10} f_{\gamma_c}(\gamma) d\gamma \quad (19)$$

Using the expression of the CDF of GSC given by (5), we get a closed-form expression for the additional average dB gain as

$$\overline{G_{\text{dB}}} = \sum_{p=0}^{\infty} \left[ \left(1 - \sum_{l=0}^{L_c-1} \beta_l G_\delta^{l-1}p \right) - \sum_{l=1}^{L_c - L_c} B_k e^{-\gamma_T/G_{\text{max}}} G_\delta^{-l}p \right] G_{\text{dB}}, \quad (20)$$

where

$$\beta_l = \frac{A_l \gamma_T^l}{\gamma_T + 1} \quad (21)$$

4) Discrete Adaptation with Saturation:

- Option 1

Using the mode of operation of PCPC, and knowing that for the discrete adaptation with saturation option 1 power control is only activated if $\gamma_c \in [\gamma_T/G_{\text{max}}, \gamma_T]$ the average BER can be shown to be given by

$$\overline{\text{BER}}_1 = \sum_{k=0}^{K_M} \int_{\gamma_T}^{10^{G_{\text{dB}} - k G_{\text{dB}}/10}} \text{BER}\left(10^{G_{\text{dB}}/10}\right) f_{\gamma_c}(\gamma) d\gamma$$

$$+ \int_{\gamma_T/G_{\text{max}}}^{\infty} \text{BER}(\gamma) f_{\gamma_c}(\gamma) d\gamma$$

$$+ \int_{\gamma_T}^{\infty} \text{BER}(\gamma) f_{\gamma_c}(\gamma) d\gamma, \quad (22)$$

where

$$K_M = \frac{G_{\text{max}} G_{\text{dB}}}{G_{\text{dB}}}. \quad (23)$$

The corresponding additional average dB gain expression is given by

$$\overline{G_{\text{dB}}} = \sum_{k=0}^{K_M} \frac{k \frac{G_{\text{dB}}}{10} f_{\gamma_c}(\gamma) d\gamma \quad (24)}$$

$$- K_M \frac{G_{\text{dB}}}{10} f_{\gamma_c}(\gamma) d\gamma \quad (24)$$

$$+ \sum_{k=0}^{K_M - 1} \int_{\gamma_T}^{10^{G_{\text{dB}} - k G_{\text{dB}}/10}} \text{BER}\left(10^{G_{\text{dB}}/10}\right) f_{\gamma_c}(\gamma) d\gamma$$
Using the expression of the CDF of GSC given by (5) in (24), the desired closed-form of $\overline{G}_{\text{dB1}}$ is shown to be given by

$$\overline{G}_{\text{dB1}} = G_{\text{max,dB}} \left( \sum_{\ell=0}^{L_c-1} \beta_\ell G_\delta^{-\ell} K_M \right) \right) = \sum_{k=1}^{K_M} \int_{0}^{T} \text{BER} \left( 10^{-\frac{k - G_{\text{dB}, k}}{10}} \right) f_{\gamma_c}(\gamma) \, d\gamma + \int_{0}^{T} \text{BER}(\gamma) \frac{f_{\gamma_c}(\gamma/G_{\text{max}})}{G_{\text{max}}(\gamma)} \, d\gamma + \int_{\gamma_T}^{\infty} \text{BER}(\gamma) \frac{f_{\gamma_c}(\gamma/G_{\text{max}})}{G_{\text{max}}(\gamma)} \, d\gamma. \tag{26}$$

The corresponding additional average dB gain is easily obtained from the result derived in option 1 and is given by

$$\overline{G}_{\text{dB1}} = G_{\text{max,dB}} + G_{\text{max,dB}} F_{\gamma_c}(\gamma_T/G_{\text{max}}) \left( \sum_{k=0}^{K_M-1} P_{\gamma_c} \left( 10^{-\frac{k - G_{\text{dB}, k}}{10}} \right) \right) G_{\text{dB}}. \tag{27}$$

A closed-form of $\overline{G}_{\text{dB}}$ is obtained in similar fashion (25) was obtained and is given by

$$\overline{G}_{\text{dB}} = G_{\text{max,dB}} - G_{\delta_{\text{dB}}} \sum_{p=0}^{L_c-1} \left( \sum_{\ell=0}^{L_c-1} \beta_\ell G_\delta^{-\ell} \right) \sum_{k=1}^{L_c} B_k e^{-\frac{k - G_\delta}{10} G_\delta^p} e^{-\frac{T}{G_\delta}}. \tag{28}$$

**B. Joint Combining and Power Control**

From the mode of operation of MS-GSC [9], we can see that the events $\Gamma_i = \sum_{j=1}^{i} \gamma_{j,L}$ are disjoint and mutually exclusive (i.e., the partial sum of the first i order statistics). By applying the total probability theorem, we can write the CDF of G as

$$F_G(g) = \text{Pr}[G \leq g] = \sum_{i=1}^{L_c} \text{Pr}[G = \gamma_T \Gamma_i \& G \leq g]. \tag{29}$$

Applying the mode of operation of JCPC as summarized in Fig. 1, we have

$$F_G(g) = \text{Pr}[\gamma_{1:L} \geq \gamma_T \& G = \gamma_T \frac{\gamma_T}{\Gamma_1} \leq g] + \sum_{i=2}^{L_c} \text{Pr}\left[\Gamma_{i-1} < \gamma_T \& \Gamma_i \geq \gamma_T \& G = \gamma_T \frac{\gamma_T}{\Gamma_i} \leq g\right] + \text{Pr}\left[\Gamma_{L_c} < \gamma_T \& G = \gamma_T \frac{\gamma_T}{\Gamma_{L_c}} \leq g\right]$$

$$= \text{Pr}[\gamma_{1:L} \geq \gamma_T \& \Gamma_{1:L} \geq \frac{\gamma_T}{\Gamma_i}] + \sum_{i=2}^{L_c} \text{Pr}\left[\Gamma_{i-1} < \gamma_T \& \Gamma_i \geq \gamma_T \& \Gamma_i \geq \gamma_T \frac{\gamma_T}{\Gamma_i} \leq g\right] + \text{Pr}\left[\Gamma_{L_c} < \gamma_T \& \Gamma_{L_c} \geq \gamma_T \frac{\gamma_T}{\Gamma_{L_c}} \leq g\right]. \tag{30}$$

Here we have to study two different cases, ($g \geq 1$ and $g < 1$), for which the CDF is then given by

$$F_G(g) = \begin{cases} 1 - (1 - e^{-\gamma_T/T})^{L_c} + \sum_{i=2}^{L_c} \text{Pr}[\Gamma_{i-1} < \gamma_T, \Gamma_i \geq \gamma_T], & g \geq 1; \\ 1 - (1 - e^{-\gamma_T/T})^{L_c} + \sum_{i=2}^{L_c} \text{Pr}[\Gamma_{i-1} < \gamma_T, \Gamma_i \geq \gamma_T], & g < 1. \end{cases} \tag{31}$$

Applying some basic probability relations we can write

$$\text{Pr}[\Gamma_{i-1} < x, \Gamma_i \geq y] = F_{\Gamma_{i-1}}(x) - F_{\Gamma_{i-1}}(y), \quad y \geq x \geq 0, \tag{32}$$

where $F_{\Gamma_{i-1}}(\cdot, \cdot)$ denotes the joint CDF of two partial sums with consecutive order statistics $\Gamma_{i-1}$ and $\Gamma_i$.

A closed-form expression for $F_{\Gamma_{i-1}}(\cdot, \cdot)$ is derived in Appendix A and is given by

$$F_{\Gamma_{i-1}}(x, y) = \sum_{j=1}^{L-1} \frac{(-1)^j L!}{(L - i - j)! i! j!} \left[ \sum_{k=0}^{i-2} (-1)^k \binom{k}{j} \right]^{k+1} \frac{1 - \left\{ 1 - \sum_{p=0}^{i-k-2} \left( \frac{x^p p!}{p!} e^{-x/p} \right) \right\}}{i - 1} e^{-(i-1) y} l(x, y) \right\}$$

$$+ (-1)^{i-1} \left\{ \frac{i-1}{j} \frac{i}{i+j} \left[ 1 - e^{-\left(\frac{i}{i+j} x\right)} \right] \right\} + \frac{L!}{(L-i)!} \left\{ 1 - \sum_{p=0}^{i-1} \left( \frac{x^p p!}{p!} e^{-x/p} \right) \right\} \right\}. \tag{33}$$
expression for $F_C(g)$ as

$$F_C(g) = \left(1 - e^{-\gamma T/g}\right)^L + \sum_{i=2}^{L_c} \sum_{i=1}^{L-1} \frac{(-1)^{i+1}}{i!} \left(\frac{\gamma T}{g}\right)^{i-1} \left(\frac{i}{\gamma T}ight)^i \left(\frac{g}{\gamma T}ight)^{i-1} \left(1 + \frac{g}{\gamma T}ight)^{i-1}$$

where $f_{\Gamma_{L_c}}(.)$ denotes the PDF of GSC and which is also available in closed-form for Rayleigh fading in [3, Eq. (16)].

1) Continuous Adaptation without Saturation: The additional average dB gain can be easily shown to be given by

$$\overline{G}_{dB} = \frac{\ln(10)}{10} \int_{-\infty}^{+\infty} g_{dB} f_{G_{dB}}(g_{dB}) dg_{dB}$$

$$= \frac{\ln(10)}{10} \int_{-\infty}^{+\infty} g_{dB} 10^{\frac{g_{dB}}{10}} f_{G} \left(10^{\frac{g_{dB}}{10}}\right) dg_{dB}.$$  

2) Continuous Adaptation with Saturation: In this case we have two options:

- Option 1

For this option, $\overline{G}_{dB}$ can be shown to be given by

$$\overline{G}_{dB1} = \frac{\ln(10)}{10} \int_{-\infty}^{G_{maxdB}} g_{dB} 10^{\frac{g_{dB}}{10}} f_{G} \left(10^{\frac{g_{dB}}{10}}\right) dg_{dB}.$$  

- Option 2

For option 2 power control is also activated when $\gamma_{c, dB} < \gamma_{T_{dB}} - G_{maxdB}$ and the additional average dB gain is given by

$$\overline{G}_{dB2} = \overline{G}_{dB1} + G_{max} \Pr[G > G_{max}]$$

$$= \frac{\ln(10)}{10} \int_{-\infty}^{\gamma_{T_{dB}}} g_{dB} 10^{\frac{g_{dB}}{10}} f_{G} \left(10^{\frac{g_{dB}}{10}}\right) dg_{dB}$$

$$+ G_{maxdB} \left(1 - F_{G_{dB}}(G_{max})\right).$$  

3) Discrete Adaptation without Saturation: By applying the mode of operation of ICPC for discrete adaptation without saturation, the expression of $\overline{G}_{dB}$ can be written as

$$\overline{G}_{dB} = \sum_{k=-\infty}^{+\infty} k \left[F_G \left(10^{\frac{k C_{dB}}{10}}\right) - F_{G_{dB}} \left(10^{\frac{k G_{dB}}{10}}\right)\right].$$
After some manipulations and simplifications (39) can be rewritten as
\[
G_{dB} = \left[ \lim_{n \to +\infty} \left( n - \sum_{k = -\infty}^{n-1} F_G \left( 10^{-\frac{G_{maxdB}}{10}} \right) \right) \right] G_{dB} \tag{40}
\]

4) Discrete Adaptation with Saturation:
- Option 1
Using the fact that \( G_{maxdB} = 0 \) if \( K > K_M \), where \( K_M \) is given in (23), we obtain \( G_{dB1} \) as
\[
G_{dB1} = G_{maxdB} F_G \left( 10^{-\frac{G_{maxdB}}{10}} \right) - \left( \sum_{k = -\infty}^{K_M-1} F_G \left( 10^{-\frac{G_{maxdB}}{10}} \right) \right) G_{dB} \tag{41}
\]
- Option 2
For the second option, \( G_{dB2} \) is easily obtained from \( G_{dB1} \) and is given by
\[
G_{dB2} = G_{dB1} + G_{maxdB} \left( 1 - F_G \left( 10^{-\frac{G_{maxdB}}{10}} \right) \right) = G_{maxdB} \left( \sum_{k = -\infty}^{K_M-1} F_G \left( 10^{-\frac{G_{maxdB}}{10}} \right) \right) G_{dB} \tag{42}
\]

IV. NUMERICAL EXAMPLES

A. Post-Combining Power Control
Fig. 2 illustrates the BER improvement that is offered by PCPC (with continuous adaptation and no HPA gain saturation) over MS-GSC. This significant BER improvement comes at the expense of an additional gain that is required from the HPA, as shown in Fig. 3. However, this additional HPA gain is (i) minimal in the medium and high average SNR region and (ii) considerably lower than the offered power control gain in the low average SNR region. As an example, note from Fig. 2 that power control provides a 3 dB gain for an average BER of \( 10^{-6} \). Correspondingly from Fig. 3, we can see that this power control gain comes at the expense of no more than 0.25 dB in additional average transmission gain.

In order to study the effect of the HPA gain saturation, we compare in what follows the average BER and the additional average dB gain for the continuous adaptation, without saturation, with saturation option 1, and with saturation option 2. We can clearly see from Fig. 4 that a peak power constraint at the transmitter side leads to a certain increase in the average BER. The lowest BER corresponds to the ideal case of continuous adaptation without saturation and the highest values of BER are those of continuous adaptation with saturation option 1.
in which the power control is activated only if the combined SNR with GSC $\in \left[\gamma_T/G_{\text{maxdB}}, \gamma_T\right]$. On the other hand, we can clearly see from Fig. 5 that a peak power constraint at the transmitter side leads to a certain decrease in the average transmitter gain. We notice here that the maximum values of the gain correspond to the ideal case of continuous adaptation without saturation and exist in the lower SNR region in which the combiner does not reach the threshold $\gamma_T$ during the MS-GSC combining process.

In order to give some numerical examples for power adaptation variants accounting for the practical implementation constraints we compare in Figs. 6 and 7, the continuous and discrete power adaptation working with a step size $G_{\delta\text{dB}} = 0.5$ dB. While we can see from Fig. 6 that discrete power control has better BER performance than continuous power control especially in the low average SNR range, Fig. 7 explains this decrease in the average BER by the slightly higher average gain needed by discrete power control.

B. Joint Combining and Power Control

In Fig. 8, we make a comparison between the average BER of BPSK with the PCPC and the JCPC schemes (continuous adaptation with no HPA gain saturation). As we can see from this figure, the performance of PCPC is better than this of JCPC in term of average BER since while the average BER of JCPC remains constant (the combined SNR with power control will be equal to the pre-required threshold SNR independently of the value of the average SNR per path), the one of PCPC decreases when the average SNR per path increases. On the other hand, Fig. 9 shows that the JCPC transmitter uses less power than PCPC transmitter especially for the high SNR range. In this range, the MS-GSC combining process always reaches the required SNR. As such JCPC transmitter will send with less than the nominal power and this gives the expected negative values for the gain.

In Fig. 10 we study the effect of HPA gain saturation over the additional average dB gain. We can notice from this figure that, like the case of PCPC, a peak power constraint at the transmitter side leads to a certain decrease in the average BER, in the low SNR range. For instance we can clearly see from this figure that JCPC experiences, when the channel conditions are poor, (i.e, $\gamma$ is small compared to $\gamma_T$), the highest values for the average dB gain. On the other hand, as $\gamma$ becomes larger, that is, the channel conditions improve, the transmitter does not communicate under the nominal power like it is the case for the PCPC, but just uses the power level that yields the minimum quality of service.
constraint when Fig. 10. Reduction in the average transmitter HPA gain under peak-power HPA with JCPC and PCPC when Fig. 9. Comparison between the additional average dB gain of the transmitter 2500 IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 7, NO. 7, JULY 2008 control variants that take into account practical implementation according to channel conditions. Four power control variants the idea behind these schemes is to adapt the transmitted power controlled MS-GSC diversity combining schemes. The key We introduced in this paper some new adaptive power-controlled MS-GSC diversity combining schemes. The key idea behind these schemes is to adapt the transmitted power according to channel conditions. Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are also proposed and studied. Selected numerical examples, show that ideal continuous power control lowers the average BER of MS-GSC and makes it always meet the specified target quality of service at the expense of an increase in the transmitter gain. Additional numerical examples, show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm.

In this paper we assumed that we have an instantaneous and error free feedback channel which is rarely the case in practical situations. Based on some simulation results that we can not include in this paper because of space limitations, we found that feedback error improves the performance of the average BER at the expense of an additional HPA gain. We also found based on these simulations that an outdated channel with a correlation factor as high as 0.8 produces a minor degradation in the BER performance.

V. CONCLUSION

We introduced in this paper some new adaptive power-controlled MS-GSC diversity combining schemes. The key idea behind these schemes is to adapt the transmitted power according to channel conditions. Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are also proposed and studied. Selected numerical examples, show that ideal continuous power control lowers the average BER of MS-GSC and makes it always meet the specified target quality of service at the expense of an increase in the transmitter gain. Additional numerical examples, show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm.

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APPENDIX A

CLOSED FORM EXPRESSION FOR $F_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(x, y)$ $F_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(x, y)$ is given, for $x \geq y \geq 0$, by

$$F_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(x, y) = \int_0^x \int_0^y f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u, v) \, dv \, du$$

$$= \int_0^x \int_0^y f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u, v) \, dv \, du$$

$$= \int_0^x \int_0^y f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u - v, u) \, dv \, du, \quad (43)$$

where $f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u, v)$ denotes the joint PDF of the $i$th order statistics, $\gamma_i$ and $L$, and the partial sum of the first $i - 1$ order statistics, $\Gamma_{i-1}$. We now use an expression of $f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots$ given in [9, Eq. (12)] as

$$f_{\gamma_i L,\Gamma_{i-1},}\ldots(u, v) = \sum_{j=0}^{L-1} a_{i,j} \left( v - (i - 1) u \right)^{i-2} e^{-\left( (i+1) u \right)}$$

$$u \geq 0, \quad v \geq (i - 1)u, \quad (44)$$

where

$$a_{i,j} = \frac{(-1)^j L!}{(L-1)! (i-1)!(i-2)!}.$$  (45)

Using (43) and taking into consideration the two cases $u < y$ and $u > y$, we can write

$$F_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(x, y) = \int_0^y \int_0^x f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u, v) \, dv \, du$$

$$+ \int_0^x \int_0^y f_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(u - v, u) \, dv \, du. \quad (46)$$

Starting from this equation, using a change of variable, and calculating some integrals available in [10, Eqs. (3.351.2), (4.331.2)] yields the desired closed-form result for $F_{\Gamma_{i-1},\Gamma_{i-1},}\ldots(x, y)$ given in (33).

REFERENCES


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