Performance Analysis of RAKE Receivers with Finger Reassignment

Seyeong Choi
Dept. of Electrical & Computer Eng.
Texas A&M University
College Station, TX 77843, USA
Email: yeong@ece.tamu.edu

Mohamed-Slim Alouini, Khalid A. Qaraqe
Dept. of Electrical Eng.
Texas A&M University at Qatar
Education City, Doha, Qatar
Email: {alouini, khalid.qaraqe}@qatar.tamu.edu

Hong-Chuan Yang
Dept. of Electrical & Computer Eng.
University of Victoria
BC, V8W 3P6, Canada
Email: hyang@ece.uvic.ca

Abstract—We propose and analyze in this paper a new finger assignment technique that is applicable for RAKE receivers when they operate in the soft handover (SHO) region. This scheme employs a new version of generalized selection combining (GSC). More specifically, in the SHO region, the receiver uses by default only the strongest paths from the serving base station (BS) and only when the combined signal-to-noise ratio (SNR) falls below a certain pre-determined threshold, the receiver uses more resolvable paths from the target BS to improve the performance. Hence, relying on some recent results on order statistics, we attack the statistics of two correlated GSC stages and provide the closed-form expressions for the SNR of the output SNR. By investigating the tradeoff among the error performance, the path estimation load, and the SHO overhead, we show through numerical examples that the new scheme offers commensurate performance in comparison with more complicated GSC-based diversity systems while requiring a smaller estimation load and SHO overhead.

I. INTRODUCTION

Multi-path fading is an unavoidable physical phenomenon that affects considerably wireless communication systems especially the wideband ones. While this phenomenon can be viewed as a deteriorating factor, it can be exploited to improve the performance by using RAKE type of receivers [1, Section 9.5.1]. These receivers use several baseband correlators called fingers to individually process multi-path signal components. The outputs from the different correlators are coherently combined to achieve improved reliability and performance.

In wideband code division multiple access (WCDMA) systems and ultra wideband (UWB) systems, the diversity branches correspond to the different resolvable multi-paths and RAKE reception is used to combine these paths. If there are \( j \) resolvable paths, the optimal number of fingers is \( j \), but due to receiver complexity and processing power constraints (especially for mobile units), we assume that \( i \) (\( \leq j \)) fingers are employed by the RAKE receiver. Usually, the mobile unit receiver is limited to 3 fingers while the base station (BS) receiver can use 4 or 5 fingers depending on the equipment manufacturer [2]. Note that in the handover (HO) region the number of available resolvable paths can be quite high since they can come from the serving BS as well as the target BS. Hence, it is natural to consider how to judiciously select a subset of paths for RAKE reception in the soft HO (SHO) region in order for the receivers to achieve the required performance while (i) maintaining a low complexity and low processing power consumption and (ii) using a minimal amount of additional network resources.

Many newly proposed low complexity combining approaches can be used for our problem of interest (i.e., combining in the SHO region) [3]–[13]. Among them is generalized selection combining (GSC) [3]–[7] which is a generalization of selection combining (SC) and which chooses a fixed number of paths with the largest instantaneous signal-to-noise ratio (SNR) from all available diversity paths and then combines them as per the rules of maximal ratio combining (MRC). This scheme offers less complexity than conventional MRC and better performance than SC. As a power-saving implementation of GSC, minimum selection GSC (MS-GSC) [8]–[10], minimum estimation and combining GSC (MEC-GSC) [11], and output-threshold GSC (OT-GSC) [12], [13] were recently proposed. With MS-GSC, after examining and ranking all available paths, the receiver tries to raise the combined SNR above a certain threshold by combining in an MRC fashion the least number of the best diversity paths. Further estimation savings can be done by using MEC-GSC. On the other hand, OT-GSC successively estimates available diversity paths and applies MRC or GSC to them in order to make the combined SNR exceed a certain SNR threshold. While these combining schemes can be applicable for our problem of interest, the way they operate does not make them distinguish the resolvable paths coming from the serving and the target BS. As such, if they are used without any modification or adaptation to the SHO, they end up using continuously the hardware/transmission resources of the serving and the target BS and result therefore in a considerable increase in overhead on the network (known as SHO overhead [14, Section 9.3.1.4]).

In this paper, we propose and study a new finger reassignment-based scheme that is specifically applicable for RAKE reception in the SHO region. With this scheme, we assume that the \( L_c \) out of total \( L \) resolvable paths from the serving BS are by default assigned to the RAKE fingers of the mobile unit in the SHO region following a predetermined fixed SNR threshold (known also as a target SNR), the receiver asks for the additional resources from the target BS. More specifically, the receiver scans the additional \( L_a \) resolvable paths from the target BS and selects again the strongest \( L_c \) paths but now among the \( L + L_a \) available

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paths (i.e., the receiver uses $L_c/(L + L_a)$-GSC). Unlike MS-GSC and OT-GSC, our proposed scheme always uses a fixed number of fingers, i.e., $L_c$, but as we will show in the performance results section, it can reduce the unnecessary path estimations and the SHO overhead compared to the conventional GSC.

The main contribution of this paper is to derive the statistics of the receiver output SNR for our newly proposed scheme, including its probability density function (PDF), cumulative density function (CDF), and moment generating function (MGF). We provide not only the analytical framework that leads to exact but complicated expressions but also an alternative approximate approach which yield relatively simple expressions that come close to the exact solutions. These results are then used (i) to analyze the performance in terms of the average probability of error and (ii) to investigate the tradeoff between complexity and performance. Some selected numerical results show that in poor channel conditions our tradeoff between complexity and performance. Some selected numerical results show that in poor channel conditions our scheme can essentially give the same performance as the GSC scheme while it offers in good channel conditions a smaller path estimation load and considerable reduction in the SHO overhead. To simplify our analysis and make it tractable, we assume that the receiver operates over a “perfect” uniform overhead. To simplify our analysis and make it tractable, we assume that the receiver operates over a “perfect” uniform propagation delay profile provided by a multi-path searcher in a way that the multi-path components are correctly assigned to the RAKE fingers.

II. FINGER REASSIGNMENT-BASED RAKE COMBINING

A. Channel and System Model

Let $\gamma_i$ denote the instantaneous received SNR of the $i$th resolvable path, $i = 1, 2, \cdots, L + L_a$. We assume that the signals from all the resolvable paths experience independent and identically distributed (i.i.d.) Rayleigh fading environments. Under a block fading assumption, the fading channel gain of each path is assumed to be constant over one time slot and vary independently from one slot to the next. As such, the faded SNR, $\gamma_i$, follows the same exponential distribution, with common PDF and CDF given as [1, Eq. (6.5)]

$$f_{\gamma_i}(x) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{x}{\overline{\gamma}}\right), \quad x \geq 0 \quad (1)$$

and

$$F_{\gamma_i}(x) = 1 - \exp\left(-\frac{x}{\overline{\gamma}}\right), \quad x \geq 0, \quad (2)$$

respectively, where $\overline{\gamma}$ is the common average faded SNR.

Next, we consider systems that employ a RAKE receiver with GSC. More specifically, we assume that, in the SHO region, there are $L$ available resolvable paths from the serving BS and $L_a$ additional available paths from the target BS, and depending on the channel conditions only $L_c$ paths among $L (L_c \leq L)$ or $L + L_a$ paths are used for RAKE reception. Now if we let $\Gamma_{i;j}$ be the sum of the $i$ largest SNRs among $j$ ones, i.e., $\Gamma_{i;j} = \sum_{k=1}^{i} \gamma_{k;j}$ where $\gamma_{k;j}$ is the $k$th order statistics (see [5] for terminology), then the total received SNR after GSC is given by $\Gamma_{L_c:L}$ or $\Gamma_{L_c:L+L_a}$.

B. Mode of Operation

Without loss of generality, we assume that at first the receiver relies only on $L$ resolvable paths gathered from the serving BS and as such starts with $L_c/L$-GSC. At the beginning of every time slot, the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used. If the receiver compares the received SNR, $\Gamma_{L_c:L}$, with a certain target SNR, denoted by $\gamma_T$. If $\Gamma_{L_c:L}$ is greater than or equal to $\gamma_T$, a one-way SHO is used.

III. STATISTICS OF THE COMBINED SNR

Although the mode of operation in (3) describes a scheme that essentially switches between $L_c/L$-GSC and $L_c/(L + L_a)$-GSC depending on the channel conditions, we can not obtain the statistics of $\gamma_t$ directly from the statistics of the output SNR with conventional GSC. Hence, in this section, we rely on some recent results on order statistics [9, 13] to derive the statistics of the combined SNR, $\gamma_t$.

Because of space limitations, we only present in this paper some of the final results. The specific details behind the derivations can be found in the journal version [15].

A. CDF

From (3), the CDF of $\gamma_t$, $F_{\gamma_t}(x)$, can be written as

$$F_{\gamma_t}(x) = \begin{cases} 
\Pr[\Gamma_{L_c:L+L_a} < x], & 0 \leq x < \gamma_T; \\
\Pr[\gamma_T \leq \Gamma_{L_c:L} < x] + \Pr[\Gamma_{L_c:L+L_a} < \gamma_T] \\
+ \Pr[\gamma_T \leq \Gamma_{L_c:L+L_a} < x], & \Gamma_{L_c:L} < \gamma_T, \quad x \geq \gamma_T.
\end{cases} \quad (4)$$

Even though the joint probability in (4) is available in closed-form, the resulting expressions are complicated and quite tedious to obtain. Here, we rather use in what follows another approximate approach which leads to results that are very close to the exact solutions as we will demonstrate it by computer simulations in Section IV. Based on the derivation in [15, Appendix], we can rewrite (4) as

$$F_{\gamma_t}(x) = \begin{cases} 
\Pr[\Gamma_{L_c:L+L_a} < x], & 0 \leq x < \gamma_T; \\
\Pr[\gamma_T \leq \Gamma_{L_c:L} < x] + \Pr[\Gamma_{L_c:L+L_a} < \gamma_T] \\
+ \Pr[\gamma_T \leq \Gamma_{L_c:L+L_a} < x] - \frac{1}{1-\Pr[\Gamma_{L_c:L} < \gamma_T]} \\
\times \Pr[\gamma_T \leq \Gamma_{L_c:L+L_a} < x] - f(x), & x \geq \gamma_T,
\end{cases} \quad (5)$$

1One-way SHO refers to the scenario in which the mobile unit is connected only to the serving BS while being in the SHO region.

2Two-way SHO refers to the scenario in which the mobile unit is connected to the serving and the target BSs while being in the SHO region.
where

\[ J(x) = \left( e^{-\frac{\gamma}{\gamma T}} - e^{-\frac{x}{\gamma T}} \right) \left( \frac{\gamma}{\gamma T} \right) \sum_{i=0}^{L_c-L_a-1} \sum_{u=0}^{L_a-1} (-1)^{i+u} \binom{L_a + L_c - L - 1}{L_a - u} \left( \frac{L - u}{\gamma T} \right)^{u+1} \times \frac{1}{(L_c - u - 1)!} \left( \frac{t+1}{\gamma T} \right)^u \cdot \frac{1}{v!} \]

(6)

where \( A_{1,a_2,\ldots,a_n} = \binom{A}{a_1,a_2,\ldots,a_n} \) is the multinomial coefficient, defined as

\[ A_{1,a_2,\ldots,a_n} = \frac{A!}{a_1!a_2!\cdots a_n!} \] with \( A = \sum_{w=1}^{n} a_w \). Since for i.i.d. Rayleigh fading channels, all other probabilities, \( P_r[\gamma] \), in (5) can be easily obtained by using the well-known CDF of the GSC output SNR [16, Eq. (9.440)], we can obtain the closed-form expression for the CDF of \( \gamma_L \) by substituting (6) in (5).

B. PDF

Differentiation of (5) gives the PDF of \( \gamma_L \), \( f_{\gamma_L}(x) \), as

\[ f_{\gamma_L}(x) = \begin{cases} f_{1-L-a-L_a}(x), & 0 \leq x < \gamma_T; \\
\frac{1}{1-F_{1-L-a-L_a}(\gamma_T)} \times (f_{1-L-a-L_a}(x) - \mathcal{I}(x)), & x \geq \gamma_T, \end{cases} \]

(7)

where \( \mathcal{I}(x) = \frac{dx}{dx} J(x) \). For i.i.d. Rayleigh fading channels, \( f_{1-L-a-L_a}(x) \) and \( F_{1-L-a-L_a}(x) \) are the well-known PDF and CDF of \( i/j \)-GSC output SNR which can be found in [16, Eqs. (9.433)(9.440)], respectively.

C. MGF

With the PDF of (7) in hand, the MGF of \( \gamma_L \), \( \mathcal{M}_{\gamma_L}(s) = \int_{0}^{\infty} e^{sx} f_{\gamma_L}(x)dx \), can be obtained in closed-form after lengthy and tedious calculations [15].

IV. PERFORMANCE RESULTS

In this section, we apply the closed-form results from the previous section for the performance analysis of our proposed combining scheme over Rayleigh fading channels. More specifically, we first examine its average bit error rate (BER) by using the well-known MGF-based approach [16, Sec. 9.2.3]. We then look into the average number of path estimations and the SHO overhead it requires.

A. Average BER Comparison with MRC and GSC

First, we consider the relationship between the number of resolvable paths from the serving BS and the average BER performance. In Fig. 1, the average BER of binary phase shift keying (BPSK) versus the average SNR per path, \( \gamma \), of the proposed scheme for various values of \( L \) over i.i.d. Rayleigh fading channels is plotted. For comparison purpose, we also plot the average BER of BPSK with \( L_c \)-MRC and \( L_c/(L + L_a) \)-GSC. In this graph, we set \( L_c = 3, L_a = 2 \), and \( \gamma_T = 5 \) dB. The simulation result for the case of \( L = 4 \) shows that our alternative simple approach is indeed a good approximation. It is clear from this figure that our proposed scheme always outperforms MRC. Also it is very interesting to note that when the channel condition is poor, i.e., \( \gamma \) is relatively small compared to \( \gamma_T \), our scheme has the same error performance as GSC. This behavior can be explained as follows. When \( \gamma \) is small compared to \( \gamma_T \), our proposed scheme acts most of the times as \( L_c/(L + L_a) \)-GSC since \( L_c/L \)-GSC output SNR has a high chance of not exceeding the required target SNR. On the other hand, in good channel conditions, our scheme shows a higher error probability. This is because when \( \gamma \) becomes larger, the combined SNR of \( L_c/L \)-GSC has a higher chance to exceed the target SNR, \( \gamma_T \), and as such does not need to rely on the additional resolvable paths from the target BS. Hence, we can conclude that our proposed combiner relies on the additional resources provided by the target BS only in poor channel conditions. For a better understanding of our scheme, we study when \( L \) is fixed and \( L_a \) is variable in what follows.

Fig. 2 shows the average BER of BPSK with MRC, GSC, and the proposed combining scheme versus the average SNR per path, \( \gamma \), for various values of \( L_a \) over i.i.d. Rayleigh fading channels when \( L = 4, L_a = 3 \), and \( \gamma_T = 5 \) dB. Similar trends to those observed in Fig. 1 can also be seen in this figure, but since \( L \) is fixed, as one expects intuitively, all the curves of our proposed scheme are converging to the case of \( L_c/4 \)-GSC in the higher average SNR region.

We now study the average BER dependence on the threshold SNR, \( \gamma_T \). Fig. 3 represents the average BER of BPSK versus the average SNR per path, \( \gamma \), with MRC, GSC, and the proposed scheme for various values of \( \gamma_T \) over i.i.d. Rayleigh fading channels when \( L = 4, L_a = 3 \), and \( L_a = 2 \). From this figure, it is clear that the higher the threshold, the better the performance, as one expects. However, high thresholds increase the path estimation load. We examine in what follows this issue in details.

B. Average Number of Path Estimations

With the proposed scheme, the RAKE receiver estimates \( L \) paths in the case of \( \Gamma_{1-L-a-L_a} \geq \gamma_T \) or \( L + L_a \) in the case of \( \Gamma_{1-L-a-L_a} T < \gamma_T \). Hence, we can easily quantify the average number of path estimations, denoted by \( N_E \), as

\[ N_E = L \cdot \Pr[\Gamma_{1-L-a-L_a} \geq \gamma_T] + (L + L_a) \cdot \Pr[\Gamma_{1-L-a-L_a} < \gamma_T], \]

(8)

which reduces to

\[ N_E = L + L_a \cdot F_{\Gamma_{1-L-a-L_a}}(\gamma_T). \]

(9)

Note that \( L_c \)-MRC and \( L_c/(L + L_a) \)-GSC always require \( L_c \) and \( L + L_a \) estimations, respectively. Fig. 4 shows the average number of path estimations versus the output threshold, \( \gamma_T \), with MRC, GSC, and the proposed scheme for various values of \( L_a \) over i.i.d. Rayleigh fading channels when \( L = 4 \),

3We note that all other numerical evaluations obtained from the analytical results derived in this paper have been also compared by Monte Carlo simulations of the system under consideration in order to justify our analytical approach.
where $\gamma = 0$ dB. For a better illustration of the trade-off between complexity and performance, Fig. 5 shows the average BER of BPSK versus the output threshold, $\gamma_T$, with MRC, GSC, and the proposed scheme. As we can see, the error rate of the proposed scheme decreases to that of $L_c/(L + L_a)$-GSC when the output threshold increases. Considering Figs. 4 and 5 together, we observe that the proposed scheme can save a certain amount of estimation load with a slight performance loss compared to GSC if the required threshold is 2 to 6 dB above $\gamma$ for our chosen set of parameters.

C. SHO Overhead

In this section, we investigate the probability of the SHO attempt and the SHO overhead. In our proposed scheme, the SHO is attempted whenever $\Gamma_{L_c:L}$ is below $\gamma_T$. Hence, the probability of the SHO attempt is same as the outage probability of $L_c/L$-GSC evaluated at $\gamma_T$, i.e., $F_{\Gamma_{L_c:L}}(\gamma_T)$. The SHO overhead, denoted by $\beta$, is commonly used to quantify the SHO activity in a network and is defined as [14, Eq. (9.2)]

$$\beta = \sum_{n=1}^{N} nP_n - 1,$$

where $N$ is the number of active BSs and $P_n$ is the average probability that the mobile unit uses $n$-way SHO. Based on the mode of operation in Section II-B, $P_1$ and $P_2$ can be defined as

$$P_1 = \Pr[\Gamma_{L_c:L} \geq \gamma_T] + \Pr[\Gamma_{L_c:L} < \gamma_T, \Gamma_{L_c:L} \geq \gamma_1:L_a],$$

and

$$P_2 = \Pr[\Gamma_{L_c:L} < \gamma_T, \Gamma_{L_c:L} < \gamma_1:L_a],$$

where $\gamma_1:L_a$ is the $L_c$th strongest path among $L$ ones from the serving BS and $\gamma_1:L_a$ is the strongest path among $L_a$ ones from the target BS. Substituting (11) and (12) into (10), we can express the SHO overhead, $\beta$, as

$$\beta = P_1 + 2P_2 - 1 = F_{\Gamma_{L_c:L}}(\gamma_T) \Pr[\Gamma_{L_c:L} < \gamma_1:L_a | \Gamma_{L_c:L} < \gamma_T].$$

With the help of the results on order statistics in [9] and [13], the conditional probability in (13) can be obtained in closed-form [15].

Fig. 6 shows the SHO overhead versus the output threshold, $\gamma_T$, of the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels when $L = 4$, $L_c = 3$, and $\gamma = 0$ dB. Simulation results are also plotted to verify our analysis. It is clear that we have a higher chance to use 2-way SHO, as the number of additional paths from the target BS increases. From this figure together with Fig. 5, we can see the SHO overhead reduction of our proposed scheme. For example, if the required threshold is 6 dB above $\gamma$, our scheme shows for $L_a = 2$ around 55% of the maximum SHO overhead while maintaining the same error rate as GSC (which requires 100% SHO overhead).

V. Conclusion

In this paper, we proposed a new finger assignment scheme for RAKE receivers in the SHO region. In this scheme, the receiver checks the GSC output SNR from the serving BS against a certain pre-determined output threshold. If the output SNR is below this threshold, the receiver performs a finger reassignment after using GSC on the paths coming from the serving BS and the target BS. We derived the statistics of the output SNR of the proposed scheme, based on which we carried out the performance analysis of the resulting systems. We showed through numerical examples that the new scheme offers commensurate performance in comparison with more complicated GSC-based diversity systems while requiring a smaller estimation load and SHO overhead.

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Fig. 1. Average BER of BPSK versus the average SNR per path, $\bar{\gamma}$, with MRC, GSC, and the proposed scheme for various values of $L$ over i.i.d. Rayleigh fading channels when $L_c = 3$, $L_a = 2$, and $\gamma_T = 5$ dB.

Fig. 2. Average BER of BPSK versus the average SNR per path, $\bar{\gamma}$, with MRC, GSC, and the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels when $L = 4$, $L_c = 3$, and $\gamma_T = 5$ dB.

Fig. 3. Average BER of BPSK versus the average SNR per path, $\bar{\gamma}$, with MRC, GSC, and the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels when $L = 4$, $L_c = 3$, and $L_a = 2$.

Fig. 4. Average number of path estimations versus the output threshold, $\gamma_T$, with MRC, GSC, and the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels when $L = 4$, $L_c = 3$, and $\bar{\gamma} = 0$ dB.

Fig. 5. Average BER of BPSK versus the output threshold, $\gamma_T$, with MRC, GSC, and the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels with $L = 4$, $L_c = 3$, and $\gamma = 0$ dB.

Fig. 6. SHO overhead versus the output threshold, $\gamma_T$, of the proposed scheme for various values of $L_a$ over i.i.d. Rayleigh fading channels with $L = 4$, $L_c = 3$, and $\bar{\gamma} = 0$ dB.