Joint Minimum Selection GSC and Down-Link Power Control*  

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Abstract—We propose and analyze in this paper a new adaptive power controlled minimum selection generalized selection combining (MS-GSC) diversity scheme termed joint combining and power control (JCPC). The key idea is to meet the required quality of service while using the minimum possible transmit and processing power. Four power control variants accounting for practical implementation constraints including discrete power levels and transmitter gain saturation are proposed and studied. Selected numerical examples show that ideal continuous power control lowers the average bit error rate of MS-GSC for poor channel conditions and makes it always meet the specified target quality of service at the expense of an increase in the transmitter gain. For good channel conditions, JCPC reduces considerably the transmitted power relative to the nominal power. Additional numerical examples, show that the power control variants that take into account practical implementation constraints conserve the main features of the ideal continuous power algorithm.

Keywords—Fading channels, Diversity Combining, and Power Control.

I. INTRODUCTION

Diversity combining is a classical concept which has been used for the past half century to combat the effects of fading on wireless systems. Over the last decade, low-complexity diversity combining schemes operating in a diversity rich environment received a great deal of attention. Among these schemes, generalized selection combining (GSC), also known as hybrid selection/maximum ratio combining (H-S/MRC), was the first to be proposed (e.g. [1]–[5]). With GSC, the $L_c$ (among the $L$ available) diversity branches with the best quality (quantified for example in terms of fading amplitude or equivalently signal-to-noise ratio (SNR)) are selected and combined as per the rules of maximal-ratio combining. Subsequently and as a power-saving implementation of GSC in the power/size limited mobile units (MUs), minimum selection GSC (MS-GSC) was proposed in [6] and recently further studied and analyzed in [7]–[9]. With MS-GSC the receiver ranks the SNR of all available paths and then combines the minimum number of branches (up to $L_c$) in order to make the combined SNR exceed a certain predetermined threshold. While the MS-GSC receiver is still constituted of $L_c$ branches (like GSC), these branches do not need to be always active, and as such MS-GSC can save (in an average sense) a considerable amount of processing power and increase the valuable battery time of the MUs. However, in very adverse channel conditions, it can happen that MS-GSC fails to guarantee the minimum required quality of service in the case the combined SNR of the $L_c$ strongest diversity branches does not exceed the predetermined SNR threshold.

In order to improve the performance of MS-GSC, post-combining power control (PCPC) scheme was proposed and studied in [10]. More specifically, numerical results in [10] showed that PCPC lowers the average bit error rate of MS-GSC and makes it always meet the specified target quality of service at the expense of an increase in the transmitter gain. In this paper, we propose joint combining and power control (JCPC), as an extension of the PCPC scheme, with the objective to reduce the transmitter gain. In particular, JCPC uses the features of MS-GSC as proposed for MUs in [6] with some downlink power control from the base station (BS) to the MU. Similar to PCPC, JCPC starts the combining process in an MS-GSC fashion and under the nominal transmitted power from the BS$^1$. If it turns out that the combined SNR of the $L_c$ strongest diversity paths still fails to exceed the predetermined SNR threshold, then the receiver orders the transmitter (through a feedback channel) to increase its transmitted power by the amount that is needed to make the combined SNR reach this required threshold. The difference between PCPC and JCPC is that while power control is activated only at the end of the combining process for PCPC, it is used simultaneously with the combining process for JCPC. More specifically, whenever the JCPC combined SNR exceeds the threshold SNR during the MS-GSC combining process, the receiver asks the transmitter to reduce its gain so that communication is established with the minimum power level that offers the required quality of service.

The remainder of the paper is organized as follows. Section II presents first the system and channel models then gives the details behind the mode of operation of the various variants of JCPC. Next, we study in section III, the statistics of the average additional gain with JCPC. These results are applied in section IV to analyze the performance of JCPC over Rayleigh fading channels. Finally, section V offers some selected numerical examples illustrating the performance of the various variants of JCPC and comparing it to the performance of PCPC.

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$^1$The BS nominal transmitted power is assumed to correspond to an initial level of output power that is adjusted/set to minimize the average outer cell interference in a particular deployment.
II. Models and Mode of Operation

A. System Model

We consider a generic diversity system with \( L \) available diversity paths. This includes for example, RAKE receivers which are used in wideband CDMA systems to combine the available resolvable multipaths. For hardware complexity considerations, we assume that up to \( L_c \) branches (\( L_c \leq L \)) can be combined at the receiver side (i.e., the number of fingers of the RAKE receiver is limited to \( L_c \)). We also assume, that the proposed JCPC scheme has a reliable feedback path between the receiver and the transmitter and are implemented in a discrete-time fashion. More specifically, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the receiver performs a series of operation, including (i) path estimation, (ii) combined SNR comparison with respect to the predetermined SNR threshold, and (iii) when needed request to the transmitter high power amplifier (HPA) to increase or decrease its gain by a specific amount. Once the suitable paths for combining are selected and once the appropriate transmitted power is reached, the combiner (at the receiver end) and the HPA (at the transmitter) are configured accordingly and this transmitter and receiver settings are used throughout the subsequent data burst.

B. Channel Model

We denote by \( \gamma_i \) (\( i = 1, 2, \ldots, L \)), the received SNR of the \( i \)th diversity path (under nominal transmitted power from the BS) and we adopt a block flat fading channel model. More specifically, assuming slowly-varying fading conditions, the different diversity paths experience roughly the same fading conditions (or equivalently the same SNR) during the data burst and its preceding guard period. In addition, the fading conditions are assumed to (i) be independent across the diversity paths and between different guard period and data burst pairs, and (ii) follow anyone of the popular fading models such as Rayleigh, Rice, or Nakagami-m.

For our study, we assume that the multipath envelop of each path follows the Rayleigh fading model. We also assume that the fading signal envelops on all diversity branches are mutually independent and identically distributed. The PDF \( f_{\gamma_i}(x) \) and CDF \( F_{\gamma_i}(x) \) of the faded SNR, denoted by \( \gamma_{i}, i = 1, \ldots, L \), for a diversity path for Rayleigh fading model are given by

\[
 f_{\gamma_i}(x) = \frac{1}{\bar{\gamma}} \exp \left( -\frac{x}{\bar{\gamma}} \right), \quad x \geq 0
\]

and

\[
 F_{\gamma_i}(x) = 1 - \exp \left( -\frac{x}{\bar{\gamma}} \right), \quad x \geq 0,
\]

respectively, where \( \bar{\gamma} \) is the common average faded SNR.

C. Mode of Operation

At the beginning of the guard period, the transmitter high power amplifier (HPA) gain \( G \) (with respect to the nominal transmitted power) is initially set to 0 dB and based on this setting the receiver starts by estimating, ranking, then combining the diversity paths in a MS-GSC fashion. With MS-GSC, the receiver selects the least number of best paths such that the SNR of combined signal, denoted by \( \gamma_c \), is greater than a preselected output threshold, \( \gamma_T \) (see [7]-[9] for more details behind the mode of operation of MS-GSC).

For PCPC [10], the receiver starts the combining process in a MS-GSC fashion, if the required output SNR \( \gamma_T \) is reached during this initial phase, then no additional HPA gain is needed and the receiver is configured during the subsequent data burst time with the suitably selected diversity branches. If, on the other hand, the MS-GSC combiner fails to meet the \( \gamma_T \) requirement during this initial phase, the receiver activates the power control mechanism and requests the HPA to increase its gain. The difference between PCPC and JCPC is that if the required output SNR \( \gamma_T \) is reached with the second scheme during this initial phase the receiver will ask the transmitter to reduce its power by the amount \( \gamma_c - \gamma_T \), and then the transmitter will provide only the power level allowing to communicate with the minimum required quality of service. The mode of operation of the JCPC scheme is summarized in a flow-chart given in Fig. 1. In this flow-chart, \( \gamma_{i:L} \) represents the \( l \)th order statistics such that \( \gamma_{1:L} \geq \gamma_{2:L} \geq \ldots \geq \gamma_{L:L} \).

We consider in our study, four power adaptation variants:

1) Continuous Adaptation without Saturation: In this first ideal case, we assume that the gain of the HPA \( G \) can be adjusted in a continuous fashion and is not limited by any maximal value.

2) Continuous Adaptation with Saturation: In this case, we still assume that the gain of the transmitter HPA can be adjusted continuously but saturates to a certain maximal value \( G_{\text{max}} \).

3) Discrete Adaptation without Saturation: Similar to the power control algorithms that are implemented in the 3 GPP standard, we assume in this case that the HPA gain can only take discrete values. This gain can be adjusted using a binary feedback and a power control step size \( G_{\text{delta}} \).

4) Discrete Adaptation with Saturation: In this most practical case, we assume that the gain takes discrete values and saturates to a fixed maximal value \( G_{\text{max}} \).

For the continuous and discrete adaptation with saturation we have two options depending on the behavior of the HPA:

- Option 1: If the receiver asks the transmitter for more than a maximum value of \( G \) (\( G_{\text{max}} \)) the power control is not activated. It means that for this first option there is no down-link power control during extreme fading conditions for which the required output SNR \( \gamma_T \) can be met even if the maximum amplifier gain is used.

- Option 2: If the receiver asks the transmitter for more than a maximum value of \( G \) (\( G_{\text{max}} \)) the HPA will only transmit with \( G_{\text{max}} \). It means that for this second option, power control with maximum amplifier gain is used in the extreme fading conditions for which the required output SNR \( \gamma_T \) will not be met during these unfavorable period of times.

III. Statistics of the Average Additional Gain

To analyze the performance of our proposed diversity combining scheme, we need the statistical characterization of
the average additional gain. Based on a new result on order statistics, we derive generic expressions for the cumulative distribution function (CDF) and probability density function (PDF) of the average additional gain with JCPC in this section.

A. Cumulative Distribution Function

The expression of the CDF of the additional gain with JCPC can be shown to be given by

\[
F_G(g) = \begin{cases} 
1 - (1 - e^{-\gamma_T g})^L + \sum_{i=1}^{L-1} \frac{(\gamma_T g)^i}{i!} e^{-\gamma_T g} & g \geq 1; \\
1 - (1 - e^{-\gamma_T g})^L + \sum_{i=1}^{L-2} \frac{(\gamma_T g)^i}{i!} e^{-\gamma_T g} & g \leq 1, 
\end{cases}
\]

where \( \Gamma_i \) is defined by

\[
\Gamma_i = \sum_{j=1}^{i} \gamma_{j:L},
\]

where \( \gamma_{j:L} \) represents the \( j \)th order statistics such that \( \gamma_{1:L} \geq \gamma_{2:L} \geq \cdots \geq \gamma_{L:L} \).

Applying some basic probability relations we can write

\[
\Pr[\Gamma_{i-1} < x_i, \Gamma_i \geq y_i] = F_{T_{i-1}}(x_i) - F_{T_{i-1}}(y_i), \quad y \geq x \geq 0,
\]

where \( F_{T_{i-1}}(\cdot) \) denotes the joint CDF of two partial sums with consecutive order statistics \( \Gamma_{i-1} \) and \( \Gamma_i \), and which was derived in closed-form by the authors in [11, Appendix]. Using this closed-form expression in Eq. (5), Eq. (3) yields the following closed-form expression for \( F_G(g) \)

\[
F_G(g) = 1 - (1 - e^{-\gamma_T g})^L + \frac{\Gamma_{i-1}}{(\gamma_T g)^{i-1}} e^{-\gamma_T g} - \sum_{i=1}^{L-2} \frac{(\gamma_T g)^i}{i!} e^{-\gamma_T g} \tag{6}
\]

where \( \Gamma_i \) is defined by

\[
\Gamma_i = \sum_{j=1}^{i} \gamma_{j:L},
\]

where \( \gamma_{j:L} \) represents the \( j \)th order statistics such that \( \gamma_{1:L} \geq \gamma_{2:L} \geq \cdots \geq \gamma_{L:L} \).

B. Probability Density Function

By differentiating Eq. (6) with respect to \( g \), we obtain the following generic expression for the PDF of the average additional gain:

\[
f_G(g) = \frac{\Gamma_{i-1}}{\left( \frac{\gamma_T}{g} \right)^{i-1}} e^{-\gamma_T g} \tag{7}
\]

\[
L \cdot \frac{\gamma_T}{g} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1} + \sum_{i=1}^{L-2} \frac{(\gamma_T g)^i}{i!} e^{-\gamma_T g} \left( \frac{\gamma_T}{g} \right)^{i-1} = \frac{\gamma_T}{g} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

\[
\cdot \sum_{k=0}^{i-2} \frac{(-1)^k}{j!} \binom{i-1}{j} \left( \frac{\gamma_T}{g} \right)^{i-k-1} \left( i, j+1 \right) e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}
\]

\[
\cdot \left( \frac{\gamma_T}{g} \right)^{i-k-1} \left( i, j+1 \right) e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

\[
\cdot \left( 1 - e^{-\gamma_T g} \right)^{L-1} \sum_{i=1}^{L} \frac{(-1)^{i-1}}{(i-1)!} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

\[
\cdot g \cdot \left( \frac{\gamma_T}{g} \right)^{i-1} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

\[
\cdot \left( \frac{\gamma_T}{g} \right)^{i-1} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

\[
\cdot \left( \frac{\gamma_T}{g} \right)^{i-1} e^{-\gamma_T g} \left( 1 - e^{-\gamma_T g} \right)^{L-1}\]

IV. PERFORMANCE RESULTS

In this section, we apply the statistics of the average additional dB gain with JCPC which we obtained in the previous section to study the performance of JCPC over Rayleigh fading channels. In what follows, we focus on the average additional dB gain performance.

A. Continuous Adaptation without Saturation

The additional average dB gain can be easily shown to be given by

\[
\overline{G}_{\text{dB}} = \int_{-\infty}^{\infty} f_G(g) \, dg = \frac{\ln(10)}{10} \int_{-\infty}^{\infty} g f_{\text{dB}}(g - 20 \mu B) f_G\left( \frac{20 \mu B}{10} \right) \, dg_{\text{dB}}. \tag{8}
\]

B. Continuous Adaptation with Saturation

In this case we have two options:

1) Option 1: The power control is activated, in the second case, only for \( \gamma_{\text{dB}} \in \left( \gamma_{\text{TdB}} - \gamma_{\text{maxdB}}, \gamma_{\text{TdB}} \right] \), and in this case the HPA increases its gain by the amount \( G_{\text{dB}} = \gamma_{\text{TdB}} - \gamma_{\text{dB}} \in [0, G_{\text{maxdB}}] \). As such the additional average dB gain can be shown to be given by

\[
\overline{G}_{\text{dB1}} = \frac{\ln(10)}{10} \int_{-\infty}^{G_{\text{maxdB}}} g f_{\text{dB}}(g - 20 \mu B) f_G\left( \frac{20 \mu B}{10} \right) \, dg_{\text{dB}}. \tag{9}
\]
constraint at the transmitter side leads to a certain increase in saturation over the average BER and the additional average negative values for the gain.

will send with less than the nominal power and this gives always reaches the required SNR. As such JCPC transmitter SNR range. In this range, the MS-GSC combining process various power adaptation variants. We focus in our numerical examples on the average bit error rate and the additional dB gain as function of the average SNR per path.

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BER of BPSK with the PCPC and the JCPC scheme. In this examples on the average dB gain as function of the average SNR per path. We include in our figures some comparisons between JCPC and PCPC as well as between the different power adaptation variants.

In Fig. 2, we make a comparison between the average BER of BPSK with the PCPC and the JCPC scheme. In this comparison we work with continuous adaptation with no HPA gain saturation for $L = 6$, $L_c = 3$, and $\gamma_T = 10$ dB. For JCPC the average BER is given by

$$\overline{BER} = \text{BER}(\gamma_T) = \frac{1}{2} \text{erf}(\sqrt{\gamma_T}), \quad (11)$$

where $\text{BER}(\gamma_T)$ is the BER of the modulation used over an additive white Gaussian noise channel with SNR $\gamma_T$, and erf(.) is the complimentary error function given by

$$\text{erf}(x) = 1 - \text{erf}(x) = 1 - \frac{1}{\sqrt{\pi}} \int_0^x e^{-t^2/2} dt. \quad (12)$$

As we can see from Fig. 2, the performance of the PCPC scheme is better than the second JCPC scheme in term of average BER. Indeed, while the average BER of JCPC remains constant, the one of PCPC decreases when the average SNR per path increases.

On the other hand, Fig. 3 shows that the JCPC transmitter uses less power than PCPC transmitter especially for the high SNR range. In this range, the MS-GSC combining process always reaches the required SNR. As such JCPC transmitter will send with less than the nominal power and this gives negative values for the gain.

In Figs. 4 and 5 we study the effect of the HPA gain saturation over the average BER and the additional average dB gain. We can clearly see from Fig. 4 that a peak power constraint at the transmitter side leads to a certain increase in the average BER in the low SNR range.

From Fig. 5 we notice that JCPC experience, when the channel conditions are poor, (i.e., $\bar{\gamma}$ is small compared to $\gamma_T$), the highest values for the average dB gain. On the other hand, as $\bar{\gamma}$ becomes larger, that is, the channel conditions improve, the transmitter does not communicate under the nominal power like it is the case for the PCPC, but just uses the power level that yields the minimum quality of service.

Finally, we can see in Fig. 6 that the discrete power control offers a small decrease in the average BER. This came, as we can see from Fig. 7, at the expense of a slightly higher HPA gain.

V. NUMERICAL EXAMPLES

In this section, we give selected numerical examples for various power adaptation variants. We focus in our numerical experiments on the average bit error rate and the additional average dB gain as function of the average SNR per path. We include in our figures some comparisons between JCPC and PCPC as well as between the different power adaptation variants.

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Fig. 2. Average BER of BPSK versus the average SNR per path, \( \gamma \), with JCPC and PCPC when \( L = 6, L_c = 3 \), and \( \gamma_T = 10 \) dB.

Fig. 3. Comparison between the additional average dB gain of the transmitter HPA with JCPC and PCPC when \( L = 6, L_c = 3 \), and \( \gamma_T = 10 \) dB.

Fig. 4. Effect of transmitting amplifier saturation on the average BER of JCPC when \( L = 6, L_c = 4, G_M = 2 \) dB, and \( \gamma_T = 5 \) dB.

Fig. 5. Reduction in the average transmitter HPA gain under peak-power constraint when \( L = 6, L_c = 4, G_M = 2 \) dB, and \( \gamma_T = 5 \) dB.

Fig. 6. Comparison of the average BER of PCPC with discrete and continuous power control for JCPC when \( L = 6, L_c = 3 \), and \( \gamma_T = 10 \) dB.

Fig. 7. Average gain of the transmitter with discrete and continuous power control for JCPC when \( L = 6, L_c = 3 \), and \( \gamma_T = 10 \) dB.