Performance Analysis of Distributed Beamforming in a Spectrum Sharing System

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Abstract—In this paper, we consider a distributed beamforming scheme (DBF) in a spectrum sharing system where multiple secondary users share the spectrum with some licensed primary users under an interference temperature constraint. We assume that the DBF is applied at the secondary users. We first consider optimal beamforming and compare it with the user selection scheme in terms of the outage probability and bit-error rate performance metrics. Since perfect feedback is difficult to obtain, we then investigate a limited feedback DBF scheme and develop an analysis for a random vector quantization (RVQ) design algorithm. Specifically, the approximate statistics functions of the squared inner product between the optimal and quantized vectors are derived. With these statistics, we analyze the outage performance. Furthermore, the effects of channel estimation error and the number of primary users on the system performance are investigated. Finally, optimal power adaptation and co-channel interference are considered and analyzed. Numerical and simulation results are provided to illustrate our mathematical formalism and verify our analysis.

I. INTRODUCTION

The increase of wireless communications services is driving the development of new spectrum allocation policies. To satisfy the spectrum demand, cognitive radio (CR) based systems have been proposed as a potential promising solution [1]-[2]. As described in [2], there are three cognitive behaviors. In this paper, we consider the spectrum underlay. The basic idea of spectrum underlay is to allow the unlicensed users (secondary users) to share the spectrum with the licensed users (primary users) under some interference constraints. That is, the interference caused by the secondary users at the primary users should be controlled to an allowable level.

Until now, the capacity of spectrum sharing systems under peak or average interference power constraints has been widely studied, see e.g.,[3]-[6]. For instance, in [3], Ghasemi and Sousa considered the channel capacity under average or peak received power constraints for different fading channels conditions. Spectrum sharing systems with multiuser diversity were investigated in [4] where one secondary user with the best channel condition is selected for signal transmission. In [5], the authors also considered the user selection problem in spectrum sharing systems. The capacity of CR systems with partial channel state information (CSI) was analyzed in [6] and other related capacity analysis can be found in the literature of [6]. Until now, most results for CR systems focus on the capacity analysis. In this paper, our motivation is to use the bit-error rate (BER) and outage probability metrics to investigate the asymptotic performance.

On the other hand, for a multiple users network, we can apply a distributed beamforming scheme (DBF) to further improve the system performance. Until now, DBF has been well investigated in one-way or two-way wireless relay networks. For instance, the authors in [7] studied the collaborative-relay beamforming with perfect CSI and optimized the relay weights under both individual and total power constraints at the relays. Reference [8] presents not only a performance comparison between the optimal beamforming and a relay selection scheme, but it also investigates the limited feedback DBF in wireless relay networks with random quantized feedback (RVQ). The performance of optimal beamforming scheme in a two-way channel network was analyzed in [9]. However, all these references did not consider a CR environment.

Recently, applying beamforming in CR environments received some considerable interest [10]-[15], especially in terms of distributed and centralized beamforming schemes. For instance, the authors of [10] considered the joint beamforming and power allocation for multiple access channels in CR networks. Later the same authors considered a multiple-input single-output (MISO) CR network with a single secondary and a single primary transceiver where they assume that the CSI from primary user to the secondary transmitter is imperfect [11]. Their results show that the optimal beamforming vector lies in a two-dimensional space spanned by the mean of the channel vector of the link between the secondary transmitter and the primary receiver and the projection of the channel vector between the secondary transmitter and the secondary receiver. Reference [12] considered a decentralized CR network with multiple primary and secondary users where every secondary transmitter is equipped with multiple antennas.
Note that the analyses in [10]-[13] are applicable for the multiple-antennas scenarios. In addition, to realize spectrum sharing between the secondary and primary users and ensure no interference to primary users, a zero-forcing beamforming scheme with the help of multiple relays in a distributed manner was proposed in [14][15].

As mentioned above, references [10]-[13] focus on the analysis for a multiple-antenna CR network. Although [14][15] considered the DBF, their analysis is available for a zero-forcing beamforming structure developed in [16] and does not need to consider the interference constraint at the primary user. In this paper, we consider a multiple secondary users system under the peak interference power constraint and assume that beamforming is applied at the multiple secondary users, which builds up a distributed beamforming CR network. For an underlay spectrum sharing beamforming scheme, it requires the secondary transmitter to obtain both the CSI of the channel gains from the secondary transmitter to the primary and secondary receivers, respectively. The results in [14][15] are based on the assumption that both CSI are perfect. However, in practice, it is difficult to obtain perfect CSI due to delayed feedback or channel estimation errors and this results into a degradation of the system performance. Particularly, due to loose cooperation between the primary user and the secondary user, it is difficult to obtain the perfect CSI between the primary receiver and the secondary transmitter. As such, [11] has considered the partial CSI case where the secondary transmitter can know both the mean and covariance of the channel vector. But [11] only considered the imperfect channel knowledge of the link between the secondary transmitter and the primary receiver and assumed that the CSI between the secondary transmitter and the secondary receiver is perfect. In this paper, we will consider a more general case where the CSI of the two links are imperfect. Analytical results are developed to characterize the performance of the above mentioned system with perfect or quantized feedback. The objective is three folds and the detailed contributions of this paper are summarized in what follows:

1) We consider a DBF network in an underlay spectrum sharing environment with multiple single-antenna secondary users, one primary receiver, and one secondary receiver. To find the performance gap between our proposed scheme and the user selection (US) scheme discussed in [4], our objective is to analyze the diversity order and present a performance comparison. For a fair comparison, we assume that perfect CSI of the links between the secondary transmitters and the secondary receiver is sent back to the secondary transmitters to form the optimal beamformer under peak received power constraint.

2) We consider a random vector (RVQ) limited feedback beamforming scheme where the beamforming vector is randomly selected from a given codebook through a finite-rate feedback channel [17]. Specifically, based on the method used in [18], we derive the cumulative density function (CDF) of the squared inner product between the quantized and the optimal beamforming vectors. We then capitalize on this result to analyze the approximate system performance and investigate the performance loss due to quantization.

3) To investigate the effect of the number of primary users on the performance of the secondary user system, we extend our analysis to a multiple primary users network and derive the statistics of the output signal-to-noise ratio (SNR). This analysis provides some useful insights explicitly. Except for the limited feedback, channel estimation errors also can result in the imperfect CSI. Therefore, we also take the channel estimation errors into consideration and obtain the optimal beamforming vector to maximize the received SNR at the secondary receiver. Moreover, we present the average capacity analysis for the user selection scheme with optimal power adaptation. To avoid numerical root finding approaches, an approximate value for the cutoff value SNR is derived. Finally, we consider the outage performance of the DBF systems in the presence of unequal power co-channel interferers (CCI).

The rest of this paper is organized as follows. In Section II, we describe the system model and assumptions. The unlimited feedback beamforming versus user selection analysis is presented in Section III. In Section IV, we present the analysis for the limited feedback case. In Section V, the performance for the multiple primary users, the proposed system with channel estimation error, adaptive transmission for the user selection scheme, and the optimal DBF with spectrum sharing in the presence of unequal power CCI are all considered, respectively. Finally, we present some numerical examples in Section VI.

Notation: The following notations are used in this paper. Bold lower case letters represent column vectors, $(\cdot)^\dagger$ and $(\cdot)^T$ indicate the conjugate transpose and transpose, respectively. $\|\cdot\|_2$ denotes a 2-norm of a vector, $(\A)\^{-1}$ denote the inverse of a matrix $\A$ and $\Diag(v)$ denotes the diagonalization of a vector $v$, respectively. $\mathcal{E}(\cdot)$ denotes the expectation operator and $a \sim \mathcal{CN}(0,1)$ denotes a complex Gaussian distribution with zero mean and unit variance.

**II. SYSTEM MODEL**

We consider a cognitive radio network where $N$ secondary users share the same spectrum with a licensed primary user.
as shown in Fig. 1. Similar to the most models, we also adopt the interference temperature concept which only allows the secondary users whose interference power received at the primary receiver is less than a given threshold $Q$ to utilize the spectrum. Let $\alpha_i$ and $\beta_i$ denote the channels gains from the secondary transmitter to the primary receiver and secondary receiver, respectively. In [3], the authors have investigated the channel capacity of the secondary systems under the average or the peak received power constraints. Here we also adopt the peak received-power constraint $P_i|\alpha_i|^2 \leq Q$ where $P_i$ is the transmitted power of the $i$-th secondary transmitter. This constraint implies that the secondary transmitter can obtain the CSI of $\alpha_i$ and $\beta_i$. It should be noted that having access to the CSI between the secondary transmitter and the primary receiver is the basic assumption of the spectrum underlay [2, Chapter 10]. We note that [1] has provided some methods to estimate the interference temperature and the channel gains. Also, the authors in [3] have pointed out that the feedback of the interference channel gains can be performed directly by the license or indirectly through a band manager which mediates between the primary system and the cognitive system. Some other channel estimation methods can be found in [19]. In most references, the authors assume that the CSI of $\alpha_i$ is perfect. However, the assumption that the secondary transmitter has perfect CSI of $\alpha_i$ is unrealistic due to the loose cooperation between the primary user and the secondary user. Denoting the estimated channel gain as $\hat{\alpha}_i$, we model the channel as

$$\alpha_i = \hat{\alpha}_i + e_i,$$  

where $e_i$ is the error term with variance $1 - \rho^2 = \sigma_e^2$ and $\rho$ denotes the correlation coefficient between the estimated and the exact channel gain. We assume $\mathcal{E}[|\alpha_i|^2] = 1$, which implies that $\mathcal{E}[|\hat{\alpha}_i|^2] = \rho^2$, and $e_i$ is independent of $\hat{\alpha}_i$. Note that $\sigma_e^2$ reflects the accuracy of estimating CSI. For instance, for perfect channel estimation, $\rho = 1$ and as such $\sigma_e^2 = 0$.

The entire transmission procedure needs two stages. Firstly, with the CSI of $\hat{\alpha}_i$, the secondary transmitter computes the transmit power and compares it with the interference temperature $Q$ to satisfy the power constraint. If the secondary transmitter can obtain the CSI of $\beta_i$, the secondary transmitters then send the weighted signals to the secondary receiver, which built up a distributed beamforming CR network. To present a clear exposition and obtain a tractable analysis, like [3], we assume that there is no additional transmit power constraint at the secondary transmitter and just employ the maximum instantaneous transmit power $Q/|\hat{\alpha}_i|^2$ of the $i$-th secondary transmitter allowed by the primary user as the transmit power. It should be noted that considering the peak power limitation has its practical consideration due to the hardware capability as mentioned in [6][21]. Thus, our analytical results can be served as some bounds. Based on this assumption, the received signal at the secondary receiver can be modelled as

$$y = \sum_{i=1}^{N} \sqrt{\frac{Q}{|\hat{\alpha}_i|^2}} \beta_i w_i s + n, \tag{2}$$

where $w_i$ is the complex weight factor and $s$ is the transmitted signal with unit energy. To keep the power constraint, we require that $\sum_{i=1}^{N} |w_i|^2 = 1$. In (2), $n$ is the Gaussian noise with zero mean and variance 1. Let us define an equivalent channel vector $\mathbf{h} = [\beta_1/|\hat{\alpha}_1|, ..., \beta_N/|\hat{\alpha}_N|]$. The resulting SNR at the secondary receiver is given by

$$\gamma = Q \sum_{i=1}^{N} |\beta_i|^2 / |\hat{\alpha}_i|^2 = Q|\mathbf{h}|^2 = Q\mathbf{w}^\dagger \mathbf{h} \mathbf{w}, \tag{3}$$

where $\mathbf{w}$ denotes the weight vector with a length of $N$.

If the channel estimation of $\beta_i$ at the secondary receiver is error-free and the bandwidth of the feedback links is unlimited, the optimal weight vector is chosen as $\mathbf{w}^* = \mathbf{h}/\|\mathbf{h}\|_2$ in order to maximize the received SNR. Therefore, the resulting received SNR is

$$\gamma = \sum_{i=1}^{N} |\beta_i|^2 / |\hat{\alpha}_i|^2 = \sum_{i=1}^{N} \gamma_i, \tag{4}$$

where $\gamma_i$ is the received SNR between the secondary receiver and the $i$-th secondary transmitter. Since both $|\hat{\alpha}_i|^2$, $|\beta_i|^2$ are exponentially distributed random variables, the probability density function (PDF) of $\gamma_i$ is readily given by

$$f_{\gamma_i}(\gamma_i) = \int_0^\infty Q f_{|\beta_i|^2}(x) f_{|\alpha_i|^2}(x) \rho^2/(Q + \rho^2 \gamma_i^2), \gamma_i \geq 0, \tag{5}$$

where $f_{|\alpha_i|^2}(\cdot)$ and $f_{|\beta_i|^2}(\cdot)$ are the PDFs of $|\alpha_i|^2$, $|\beta_i|^2$, respectively. The corresponding CDF is given by

$$F_{\gamma_i}(\gamma_i) = 1 - Q/(Q + \rho^2 \gamma_i)^{-1}, \gamma_i \geq 0. \tag{6}$$

III. PERFECT FEEDBACK BEAMFORMING VERSUS USER SELECTION

In [4], multiuser diversity gain was investigated in spectrum sharing systems. The used performance metric was the system capacity. We focus here on the outage probability and BER performance. To get additional insight, we present a performance comparison between the user selection scheme developed in [4] and our proposed optimal DBF scheme.

A. Analysis of User Selection

For a fair comparison, we employ the maximum transmit power $Q/|\hat{\alpha}_i|^2$ for the user selection scheme. The secondary user with the maximum $\gamma_i$ will be chosen to transmit signals to the secondary receiver. According to the order statistics, the PDF of $\max_{1 \leq i \leq N} \gamma_i$ can be expressed as

$$f_{\gamma_{\max}}(\gamma) = NF_{\gamma_i}(\gamma)[F_{\gamma_i}(\gamma)]^{N-1} = \frac{NQ\rho^{2N}}{(Q + \rho^2 \gamma)^{N+1}} \gamma^{N-1}, \gamma \geq 0. \tag{7}$$

1) Outage Probability Analysis

Using (7), the system outage probability for a given threshold $\gamma_{th}$ can then be computed as

$$P_{out}^S = \Pr(\gamma_{\max} < \gamma_{th}) = NQ\rho^{2N} \int_0^{\gamma_{th}} \frac{\gamma^{N-1}}{(Q + \rho^2 \gamma)^{N+1}} d\gamma. \tag{8}$$
With the help of identity [22, Eq.(3.194.1)] and using the fact that \( \prod_{i=1}^{n} x_i = (1 - z)^{-n} \) [23], we have
\[
P_{out}^S = \frac{\beta^2 N \gamma_{th}^N}{Q N} 2F_1(N + 1, N; N + 1; -\frac{\beta^2}{Q \gamma_{th}}) = \left( \frac{\rho^2 \gamma_{th}}{Q + \rho^2 \gamma_{th}} \right)^N, \tag{9}
\]
where \( \prod_{i=1}^{n} x_i = (1 - z)^{-n} \) is the Gaussian hypergeometric function [22].

2) BER Analysis

From [24], we can evaluate the average BER of binary phase shift keying (BPSK) by using the CDF \( F_{\gamma_{max}}(\gamma) \) of \( \gamma_{max} \) by expressing it in terms of a standard normal distribution random variable \( V \), namely,
\[
P^S_b(E) = E_V \left\{ F_{\gamma_{max}} \left( \frac{V^2}{2} \right) \right\}. \tag{10}
\]
Note that \( F_{\gamma_{max}}(\gamma) \) can be directly obtained from (9) by replacing \( \gamma_{th} \) with \( \gamma \). Hence, we can evaluate the BER performance as follows
\[
P_b^S(E) = \int_0^\infty \left( \frac{\rho^2 v^2}{2Q + \rho^2 v^2} \right)^N \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv = \frac{1}{2\sqrt{\pi}} \left( \frac{Q}{\rho^2} \right)^{1/2} \Gamma(N + 0.5) \Psi \left( N + 0.5, 1.5, -\frac{Q}{\rho^2} \right), \tag{11a}
\]
where \( \Psi(a; b; z) \) is the Tricomi confluent hypergeometric function and defined in [22, Eq.(9.211.4)] and \( pF_q(a_1, \ldots, a_q; b_1, \ldots, b_q; z) \) is the generalized hypergeometric function. The equality (11b) follows from [22, Eq.(3.383.5)]. The last identity (11c) uses the fact \( F_0(\alpha, \beta; -1/x) = x^\alpha \Psi(\alpha, \alpha - \beta + 1, x) \) [25].

To investigate the diversity order of the user selection scheme, we need to find an asymptotic expression for (11). For high \( Q \), we have \( (2Q + \rho^2 v^2)^{-N} \approx \left( 2Q \right)^{-N} \). Thus, the BER in (11a) can be approximated by
\[
P_b^S(E) \approx \frac{\beta^2 N}{Q N^{2\sqrt{\pi}}} \Gamma(N + 0.5). \tag{12}
\]
With (9) and (13), we can now consider the ratio of the outage probability for the user selection and optimal beamforming schemes leading to
\[
\frac{P^S_{out}}{P^{OBF}_{out}} = \left( \frac{NQ + \rho^2 \gamma_{th}}{Q + \rho^2 \gamma_{th}} \right)^N \approx N^N, \quad Q \to \infty. \tag{14}
\]
We can observe that the optimal beamforming scheme leads to a higher system performance with the increase of the number of secondary users. However, as mentioned in [8], optimal beamforming is not practical in realistic wireless applications due to its high amount of feedback. For this reason, we will consider the RVQ limited feedback scheme in Section IV.

2) BER Analysis

Lemma 1: For i.i.d Rayleigh fading channels, the BER of a spectral sharing system with perfect beamforming feedback is given by
\[
P^{OBF}_b(E) = \frac{1}{\pi} \int_0^\pi \left( 1 - \Delta(\theta) e^{\Delta(\theta)} E_1(\Delta(\theta)) \right)^N d\theta, \tag{15}
\]
where \( \Delta(\theta) = Q g/\rho \sin^2 \theta \), \( E_1(x) \) is the exponential integral function, and \( q = 1 \) for BPSK and \( q = 0.5 \) for binary frequency-shift-keying (BFSK).

Proof: See Appendix A.

Eq.(15) can not reveal much information about the diversity order. Also, we can not directly use it to make a comparison with (12). Thus, again, we apply the bound \( \gamma \leq N \gamma_{max} \) to compute the asymptotic BER. Using the same steps that lead to (12), we can show that the approximate BER of the perfect beamforming feedback is given by
\[
P^{OBF}_b(E) \approx \frac{\rho^{2N}}{N^N Q^N 2\sqrt{\pi}} \Gamma(N + 0.5). \tag{16}
\]
From (16), we can see that the optimal DBF scheme also achieves a diversity order \( N \). In addition, looking at (12) and (16) allows us to conclude that
\[
\frac{P_b^S(E)}{P^{OBF}_b(E)} \approx N^N. \tag{17}
\]
In addition to the asymptotic formula (16), we can use the technique developed in [26] and used in [24] to derive an alternative asymptotic BER expression for (15). From (5) and (6), we observe that \( f_{\gamma_{th}}(0) \neq 0 \) and \( f_{\gamma_{th}}(0) = 0 \) which satisfies the condition required in [26]. Thus, according to [26], the asymptotic average BER expression is readily given by
\[
P^{OBF}_b(E) \to \prod_{n=1}^{N} \frac{\left( 2n - 1 \right)}{N 2^{N + 1}} (f_{\gamma_{th}}(0))^N = \prod_{n=1}^{N} \frac{\left( 2n - 1 \right) \rho^{2N}}{N 2^{N + 1} Q^N}. \tag{18}
\]
Using the fact that \( \Gamma(N + 0.5) = \frac{\pi^{1/2}}{\sqrt{2}} \prod_{n=1}^{N} \frac{\left( 2n - 1 \right)}{N 2^{N + 1}} \) [22, Eq.(8.339.2)] in (18), we finally get
\[
P^{OBF}_b(E) \approx \frac{\rho^{2N}}{N Q^N 2\sqrt{\pi}} \Gamma(N + 0.5). \tag{19}
\]
As we will show in numerical results section, (19) is a tight approximation for the exact value (15) at high SNR.
IV. ANALYSIS FOR THE RVQ LIMITED FEEDBACK

In the previous section, the analysis was based on the perfect feedback assumption where the bandwidth of the feedback link from the secondary receiver to the secondary transmitter was assumed unlimited. In this section, we assume that only a limited number of bits is sent from the secondary receiver to the secondary transmitter to indicate which beamforming vector is chosen. Since the exact performance analysis is almost impossible, we provide some bounds to approximate the outage probability which helps us to evaluate the effect of quantization on the system performance.

According to the definition in [17][18], in a RVQ scheme, the beamforming vector which maximizes the received SNR is chosen from a randomly generated codebook \( \mathcal{W} = \{w_1, ..., w_M\} \) known to both the transmitter and receiver ends. Therefore, the conditional SNR is given by

\[
\gamma = \max_{w \in \mathcal{W}} Q(|hw|^2).
\]

Notice that only the index number is conveyed to the secondary transmitter through an error-free and no-delay feedback link.

Like [17], the SNR in (20) can be rewritten as

\[
\gamma = Q X U,
\]

where \( X = \max_{w \in \mathcal{W}} |hw|^2/||h||_2^2, \quad U = ||h||_2^2. \) Under the i.i.d assumption, \( X \) is independent of \( U \).

Lemma 2: If \( h = \sum_{i=1}^{N} \beta_i/\alpha_i \) where \( \beta_i \sim CN(0, 1) \) and \( \alpha_i \sim CN(0, \rho^2) \), then the CDF of \( X \) in a spectral sharing system with RVQ limited feedback can be bounded as

\[
\left[ 1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - 2x}{1 - x} \right) \right]^M < F_X(x)
\]

\[
\left[ 1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - Nx}{1 - x} \right) \right]^M, \quad x \in (0, 1).
\]

Proof: See Appendix B.

Based on (22), we can evaluate the outage probability as shown in what follows. From (21), the system outage probability can be expressed as

\[
P_{\text{out}}^{L_i} = \Pr(Q X U \leq \gamma_{th}) = \int_0^1 F_U \left( \frac{\gamma_{th}}{Q X} \right) f_X(x) dx,
\]

where the superscript \( L_i \) denotes the limited feedback beamforming, \( F_U(u) \) is the CDF of \( U \), and \( f_X(x) \) is the PDF of \( X \). As stated before, the exact analysis for the statistics of \( U \) is difficult. Hence, similar to the analysis for the CDF of \( X \), we can obtain upper and lower bounds as

\[
[1 - N(N + \rho^2 u)^{-1}]^N < F_U(u) < [1 - (1 + \rho^2 u)^{-1}]^N.
\]

Using integration by parts, (23) can be rewritten as

\[
P_{\text{out}}^{L_i} = F_U \left( \frac{\gamma_{th}}{Q} \right) + \gamma_{th} \int_0^1 \frac{1}{x^2} F_X(x) f_U \left( \frac{\gamma_{th}}{Q x} \right) dx
\]

where \( f_U(u) \) is the PDF of \( U \). Note that the first term represents the optimal feedback case and the second term denotes the performance loss due to the quantization of the beamforming vector.

By substituting the PDF or CDF of \( U \) and \( X \) into (25), the approximate outage probability is given by

\[
\frac{\Omega^N}{(\Omega + N)^N} + N^2 \Omega^N
\]

\[
\times \int_0^1 \left[ 1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - 2x}{1 - x} \right) \right]^M dx < P_{\text{out}}^{L_i}
\]

\[
\frac{\Omega^N}{(\Omega + N)^N} + N \Omega^N
\]

\[
\times \int_0^1 \left[ 1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - Nx}{1 - x} \right) \right]^M dx,
\]

where the constant \( \Omega = \rho^2 \gamma_{th}/Q \). Unfortunately, no closed-form is available for the integrals in (26) and numerical integration is therefore needed. From (26), we can clearly see that the terms incurred by the quantization will disappear with increasing the number of beamforming vectors \( M \).

Therefore, the system performance of RVQ limited feedback beamforming approaches the optimal beamforming case when we increase the quantization levels.

Now we consider the average SNR for this system. From (21), the average SNR can be shown to be given by

\[
E[\gamma] = Q E[U] \left( 1 - \frac{1}{0^1} F_X(x) dx \right).
\]

With (22), if \( M \to \infty, F_X(x) \to 0 \) for \( x \in (0, 1) \). Then, \( E[\gamma] \to Q E[U] \). Hence, we conclude that the gap to average SNR and outage probability between the limited and unlimited feedback beamforming can be reduced by increasing the size of the codebook, namely, \( \log_2 M \) bits of feedback.

V. EFFECTS OF MULTIPLE PRIMARY USERS, CHANNEL ESTIMATION ERROR, ADAPTIVE TRANSMISSION, AND CO-CHANNEL INTERFERENCE

In the previous sections, the analysis was based on the assumptions: single primary user, no channel estimation error between the secondary transmitters and the secondary receiver, uniform power allocation among the transmitters, and no co-channel interference. In this section, we take these factors into account and present the corresponding performance analysis to investigate their effects on the system performance.
A. Effect of the Number of Primary Users on the System Performance

In this sub-section, we consider a spectral sharing system with $N_p$ primary users and assume that the channels are independent but not identically distributed (i.n.i.d). Such a spectrum sharing environment with multiple primary users can be realized in practice as shown in Fig.2 (Please refer to [2, Fig.10.11]). From Fig.2, we can see that the secondary users share the licensed frequency bands with the multiple primary users and every primary user cell network has different spectrum range. Detailed description of this system can be found in [2]. Thus, the secondary transmitter $i$ will be corrupted by $N_p$ interferers. With these assumptions, the BER will be analyzed for the optimal beamforming case.

Let $\hat{\alpha}_{ij}$ represent the estimated channel coefficient between the $i$th secondary transmitter and the $j$th primary user. Similar to the analysis in [3][4], the transmitted power of the $i$th secondary transmitter can be expressed as

$$P_i \leq \min_{|\hat{\alpha}_{ij}|^2} \frac{Q_j}{|\hat{\alpha}_{ij}|^2} = \max_{|\hat{\alpha}_{ij}|^2} \frac{Q_j}{|\hat{\alpha}_{ij}|^2}, \quad j = 1, \ldots, N_p. \quad (28)$$

As the mentioned model in Section II, the total received system SNR for optimal beamforming is given by

$$\gamma = \sum_{i=1}^{N} \frac{|\beta_i|^2}{P_i} = \sum_{i=1}^{N} \gamma_i. \quad (29)$$

For un-identical $Q_j$ and $E[|\hat{\alpha}_{ij}|^2]$, a closed-form analysis is not easily obtainable. Hence, for analytical tractability, we assume that $E[|\hat{\alpha}_{ij}|^2] = \rho_s^2$ and $Q_j = Q$ for all $j$.

According to the order statistics, the PDF of $P_i$ is readily given by

$$f_{P_i}(x) = \frac{Q_i}{\rho_i^2} N_p \left(1 - e^{-\frac{x}{\rho_i^2}}\right)^{N_p-1} e^{-\frac{x}{\rho_i^2}}, \quad x \geq 0. \quad (30)$$

Defining $E[|\beta_i|^2] = \sigma_i^2$, then, the PDF of $\gamma_i$ can be determined as

$$f_{\gamma_i}(\gamma) = \int_0^\infty x f_{|\beta_i|^2}(x) f_{P_i}(x) dx = \frac{Q_i N_p}{\sigma_i^2 \rho_i^2} \sum_{n=0}^{N_p-1} \left(\frac{\gamma}{\sigma_i^2 + \rho_i^2}\right)^{n} \left(\frac{\gamma}{\rho_i^2}\right)^{N_p-1-n}, \quad \gamma > 0. \quad (31)$$

Notice that (31) is equal to the result in [3] when we set $Q_i$, $\sigma_i^2$, and $\rho_i^2$ to 1. From (31), we see that $f_{\gamma_i}(0) \neq 0$. Therefore using the same steps that lead to (18), the asymptotic BER can be found to be given by

$$P_b^{MP}(E) \approx \prod_{i=1}^{N_p} \left(1 - \frac{1}{N!2^{N+i+1}} \sum_{n=0}^{N_p-1} \frac{\gamma}{\rho_i^2} \left(\frac{\gamma}{\sigma_i^2}\right)^{n} \left(\frac{\gamma}{\rho_i^2}\right)^{N_p-1-n}\right). \quad (32)$$

Note that (33) becomes (18) when $N_p = 1$. Now considering the ration between (18) and (33), we get

$$\frac{P_b^{MP}(E)}{P_b^{OBF}(E)} = \left(\frac{N_p}{\sum_{n=0}^{N_p-1} \left(\frac{N_p-1}{n}\right) \left(\frac{1}{(1+n)^2}\right)}\right)^N. \quad (34)$$

The ratio is bigger than 1 for all $N_p \geq 2$, for example, $P_b^{MP}(E) = 1.5 N P_b^{OBF}(E)$ for $N_p = 2$. Thus, we can conclude that the system performance decreases with the increase of number of primary users as one might expect. Although a similar observation was reported in [3][4], it has not been explicitly quantified.

B. Effect of Channel Estimation Error on the System Performance

Up to this point, all the results are based on the assumption that the estimated channels of the secondary system are error-free. In realistic environments, there exist typically errors in the estimated CSI. In this case, we need to take the errors into consideration for the design of the DBF scheme. We still assume in this section that the estimated CSI is fed back from the secondary receiver to the secondary transmitters via a delay-free feedback link which can be ensured by the auto-request-query technique (ARQ). Similar to the definition in (1), we start with the model

$$\hat{\beta}_i = \beta_i + \hat{e}_i, \quad (35)$$

where $\hat{e}_i$ is the error term with variance $1 - \hat{\rho}^2 = \hat{\sigma}_i^2$ and $\hat{\rho}$ is the correlation coefficient. We assume $E[|\hat{\beta}_i|^2] = 1$, which implies that $E[|\hat{\beta}_i|^2] = \hat{\rho}^2$. Note that $\hat{e}_i$ is independent of $\beta_i$.

For the channels with estimation error, the received signal at the secondary receiver in (2) should be rewritten as

$$y = \sum_{i=1}^{N} \sqrt{\frac{Q}{|\hat{\alpha}_i|^2}} (\hat{\beta}_i + \hat{e}_i) w_i s_n + n \quad \Rightarrow \quad \sum_{i=1}^{N} \sqrt{\frac{Q}{|\alpha_i|^2}} \hat{\beta}_i w_i s_n + \sum_{i=1}^{N} \sqrt{\frac{Q}{|\alpha_i|^2}} \hat{e}_i w_i s_n + n. \quad (36)$$

Treating the second term as noise, the conditional SNR is now given by

$$\gamma = \frac{Q \left(\sum_{i=1}^{N} \frac{\hat{\beta}_i}{|\alpha_i|^2} w_i\right)^2}{Q\hat{\sigma}_e^2 \sum_{i=1}^{N} \frac{|w_i|^2}{|\alpha_i|^2} + 1} = \frac{Q w^T \hat{h} w}{w^T (I + Q \hat{\sigma}_e^2 \hat{H}^T \hat{H}) w}. \quad (37)$$

where $\hat{h} = [\hat{\beta}_1/\alpha_1, ..., \hat{\beta}_N/\alpha_N]^T$, $\hat{H} = \text{Diag}[1/\alpha_1, ..., 1/\alpha_N]$, and $w$ is a column vector composed of $w_i$. Using the analytical method in [8], maximizing $\gamma$ with respect to $w$, we can obtain the optimal weight vector as

$$w^* = \frac{(I + Q \hat{\sigma}_e^2 \hat{H}^T \hat{H})^{-1} \hat{h}}{\| (I + Q \hat{\sigma}_e^2 \hat{H}^T \hat{H})^{-1} \hat{h} \|^2} \quad (38)$$
This maximization can also be solved by using the Cauchy-Schwartz inequality [22, Eq.(11.112)]. The resulting optimal SNR is then given by
\[
\gamma = \frac{Q \left( \sum_{i=1}^{N} \left| \tilde{h}_i w_i \right|^2 \right)^2}{Q \sigma^2 \sum_{i=1}^{N} \left| \frac{w_i}{\alpha_i} \right|^2 + 1}.
\]
and the last equality follows from the integral identity [22, Eq.(11.112)].

From (39), we can observe that the resulting SNR becomes
\[
\gamma = \frac{Q \sum_{i=1}^{N} \left| \tilde{h}_i \right|^2}{Q \sigma^2 \sum_{i=1}^{N} \left| \frac{1}{\alpha_i} \right|^2 + 1},
\]
where \(X_1 = N \max_{1 \leq i \leq N} \left| \tilde{h}_i \right|^2 \) and \(X_2 = N/\min_{1 \leq i \leq N} \left| \alpha_i \right|^2 \). As expected, when \(\tilde{\sigma}^2 = 0\), (40) reduces to (4).

Taking an analytical approach similar to the one used in the previous sections, the PDFs of \(X_1\) and \(X_2\) can be obtained as
\[
f_{X_1}(x) = N^2 \rho^2 \rho^{2N} x^{-N-1} (x \rho^2 + N \rho^2)^{-(N+1)}, \quad x \geq 0,
\]
and
\[
f_{X_2}(x) = \frac{N^2 \rho^{2N} e^{-\frac{x^2}{2 \rho^2}}}{\rho^{2N} x^2}, \quad x \geq 0.
\]

Therefore, we can simply express the outage probability performance as
\[
P_{out} = \int_0^{\infty} F_{X_1} \left( \gamma_{th} \frac{Q \sigma_x^2 x + 1}{Q} \right) f_{X_2}(x) dx.
\]
Using the equality \( \int_0^{\infty} \left( \frac{\rho^2 \gamma_{th} (Q \sigma_x^2 x + 1)}{\rho^2 \gamma_{th} (Q \sigma_x^2 x + 1) + Q N \rho^2} \right)^N \frac{N^2 \rho^{2N} e^{-\frac{x^2}{2 \rho^2}}}{\rho^{2N} x^2} \) \(N\),
\[
\left\{ \begin{array}{l}
NQ \gamma_0 + \frac{Q^2}{\gamma_0} F_1 \left( -N - 2, 1; -\frac{Q}{\gamma_0} \right) = 2\gamma_0 \end{array} \right.
\]
where \(\gamma_0 = Q \eta_0\). Generally, a numerical root finding technique is required to solve (49). However, for high \(Q\), and according to the resulting first term in the Gaussian hypergeometric function \( F_1(a, b; c; z) = 1 \) [22, Eq.(9.122.1)],
\[
\eta_0 \approx \frac{\sqrt{Q^2 + 4Q - Q}}{2Q} - \frac{1}{2},
\]
It was shown in [28] that \(\gamma_0\) satisfies \(\gamma_0 \in [0, 1] \). Note that this conclusion is also valid for our work and can be verified
with the help of [22, Eq.(9.122.1)]. More specifically, from (49), we have
\[ 2F_1 \left( -(N - 2), 1; 3; \frac{Q}{Q + \gamma_0} \right) \]
\[ \rightarrow 2F_1 \left( -(N - 2), 1; 3; 1 \right) = \frac{2}{N}, \text{ for high } Q. \]  
(51)

Then, substituting (51) into (49), we get
\[ \gamma_0 \approx \frac{Q}{\gamma_0 + Q}. \]  
(52)

From (52), we can see that \( \gamma_0 \rightarrow 1 \) as \( Q \rightarrow \infty \). Also, \( \gamma_0 \rightarrow 0 \) as \( Q \rightarrow 0 \).

Substituting the PDF of \( \eta \) into (45), the average capacity for the spectrum sharing system with user selection and optimal power adaptation can be shown to be given by
\[ C = \log_2(e) \frac{N - 1}{\eta_0} \sum_{n=0}^{N-1} \left( \frac{\eta_0}{1 + \eta_0} \right)^{N-n} \times \left[ \log_2(e) N \ln(\eta_0)/2F_1(n, \eta_0; 1, 1, 0) \right] \]
\[ - \log_2(e) N \ln(\eta_0)/2F_1(1, 1, 2; 1 - \eta_0), \]  
(53)

where \( G_{m,n}^{p,q} \) is the Meijer’s G-function [22, Eq.(9.30)]. In order to evaluate the integral in (45), the transformation of variable \( x = y + \eta_0 \) is firstly used.

Then, the log function is expressed in terms of the Meijer’s G-function [23] before [22, Eq.(7.811.5)] is used.

D. Effect of Co-channel Interference on the System Performance

In this subsection, we consider the system performance of the distributed beamforming spectrum sharing system in the presence of \( L \) co-channel interferers. Without loss of generality, we assume that the interferers might have different powers.

For simplicity of exposition, in this section we consider the perfect CSI case. Therefore, the received signal at the secondary receiver can be written as
\[ y = \sum_{i=1}^{N} \sqrt{Q/|\alpha_i|^2} \beta_i w_i s_i + \sum_{l=1}^{L} \sqrt{P_{l,i} h_{l,i} s_i} + n, \]  
(54)

where \( s_i \) and \( P_{l,i} \) are the transmitted signal and the power of the \( i \)-th interferer, respectively. \( h_{l,i} \) denotes the channel coefficient with the distribution \( \mathcal{N}(0, 1) \).

From (54), the instantaneous output signal-to-interference noise ratio (SINR) is given by
\[ \mu = \frac{Q \sum_{i=1}^{N} |\beta_i w_i|^2}{\sum_{i=1}^{L} P_{l,i} |h_{l,i}|^2 + 1}. \]  
(55)

Using Swartz inequality [22] and maximizing \( \mu \) yields \( w_i^* = \frac{|\beta_i|}{\sqrt{\sum_{i=1}^{N} |\beta_i|^2}} \), and the resulting SINR becomes
\[ \mu = \frac{Q \sum_{i=1}^{N} |\beta_i|^2}{\sum_{i=1}^{L} P_{l,i} |h_{l,i}|^2 + 1} = \frac{Q \sum_{i=1}^{N} X_i}{P_l \sum_{i=1}^{L} P_{l,i} |h_{l,i}|^2 + 1} \]
\[ = \frac{Q \sum_{i=1}^{N} X_i}{P_l \sum_{i=1}^{N} Y_i + 1}. \]  
(56)

where \( P_l \) is the total interference power and the PDF of \( X_i \) can be directly obtained from (5) by setting \( Q = \rho^2 = 1 \). \( Y_i \) are exponential random variable with parameter \( \lambda_i = P_{l,i}/P_l \) and notice that the PDF of \( Z = \sum_{l=1}^{L} Y_i \) was derived in [29][30]. Since the distribution of the sum of \( X_i \) is difficult to be obtained, we again need to use the bound \( X = N \max_{1 \leq i \leq N} X_i \) to analyze the system performance.

Hence, the outage probability can be written as
\[ P_{out} \approx \Pr \left( \frac{QX}{P_l Z + 1} < \gamma_{th} \right) \]
\[ = \int_{0}^{\infty} \Pr \left( X < \frac{\gamma_{th}(P_l Z + 1)}{Q} \right) f_Z(z) dz. \]  
(57)

Substituting the density functions of \( X \) and \( Z \) into (57), we have
\[ P_{out} \approx \sum_{i=1}^{L} \sum_{j=1}^{N} \chi_{i,j}(V) \left[ \frac{v_{ij}^{n-j}}{(j-1)!} \right] \]
\[ \times \int_{0}^{\infty} \left( \frac{\gamma_{th}(P_l Z + 1)}{Q N + \gamma_{th} (P_l Z + 1)} \right)^N \times \frac{\left( \frac{\gamma_{th}}{Q N + \gamma_{th}} \right)^N}{\sum_{i=1}^{L} \sum_{j=1}^{N} \chi_{i,j}(V) (j-1)! \sum_{n=0}^{N} (j+n)} \]
\[ \times \Gamma(j+n)(v_{ij} P_l)^n 2F_0 \left( j+n, N; -\frac{v_{ij} \gamma_{th} P_l}{Q N + \gamma_{th}} \right), \]  
(58)

where \( V \) is a diagonal matrix with the elements \( \alpha_i \) and \( \alpha(V) \) denotes the number of distinct diagonal elements. \( v_{ij} \) are the ordered values of the \( \alpha(V) \) distinct dialog elements and denoted as \( v_{ij} > v_{ij} > \cdots > v_{ij} \). \( \tau_0(V) \) represents the multiplicity of \( v_{ij} \) and \( \chi_{i,j}(V) \) is the characteristic coefficient of \( V \). Note that all the above-mentioned parameters are defined in [29][30]. The final equality follows from [22, Eq.(3.383.5)] and [25].

VI. NUMERICAL RESULTS

In this section, we present the numerical results to verify and illustrate our analysis. Unless otherwise specified, we assume that \( \rho^2 = 1 \).

The comparison for the outage probability between the user selection and our proposed DBF schemes are shown in Fig.3. The SNR threshold \( \gamma_{th} \) is set to 5 dB and \( N \) is set to 3. The analytical results are plotted by using (9) and (13). Since (13) is the lower bound of the exact value, we also present the exact result obtained by simulations. As expected, the proposed DBF scheme has better performance than the user selection scheme. We also observe that the simulation results match their corresponding analytical results perfectly.

Fig.4 shows the BER results for the optimal DBF and user selection scheme when \( N = 3 \). For the user selection case, the analytical result (11) is in agreement with the simulations. Since the analysis is (16) is a lower bound and obtained in term of \( N \gamma_{max} \), the simulation curve according to \( N \gamma_{max} \) is also plotted. Compared with their corresponding simulation results, it is clearly observed that the approximate results (12), (16) and (19) are tight at high SNR. We can see that formula (19) is a good approximation for the exact BER performance of the
The OBF scheme. Furthermore, we can observe that the proposed scheme can achieve better performance.

Fig. 5 shows the outage probability comparison between the lower and upper bounds (26) and the exact simulation results at a SNR threshold of 5 dB when $N = 3$ and $M = 4, 8$. The result for the perfect feedback beamforming is also plotted to show the quantization loss. The $M$ vectors are randomly generated and all are uniformly distributed on the complex unit-norm sphere. We observed that these bounds can reflect the scaling law of the system outage probability. In Fig.6, we plot the outage probability curves for different $M$. As expected, the system performance due to quantization is decreased. However, with the increase of the number of random vectors in the codebook, the quantized feedback scheme approaches the perfect feedback case.

Fig. 3. Outage probability comparison between perfect-feedback DBF and user selection.

Fig. 5. Outage probability comparison between the perfect-feedback and limited feedback beamforming schemes.

Fig. 4. BER comparison between perfect-feedback DBF (OBF) and user selection (US).

Fig. 6. Outage probability comparison for different values of $M$.

The effect of channel estimation error on the DBF spectrum sharing system performance when $N = 3$ and $\gamma_{th} = 2$ dB is shown in Fig. 8. We can see that the channel estimation error can reduce the system performance significantly. Thus, the effect of channel estimation error is an important issue to be considered in the design and assessment of the performance of DBF systems. Note again that, the simulation results verify the correctness of our analysis.

Fig. 7 shows the BER when $N_p = 1, 2$ and $N = 3$. The SNR threshold is set to 2 dB. From Fig.6, it is clearly shown that the number of primary users has a detrimental effect on the system performance as observed in [3][4]. The reason is that the secondary transmitters have many power constraints, which in turn reduces the transmit power of the secondary transmitters. We can see that (33) becomes tight as SNR increases.
Fig. 7. BER for a multiple primary users system.

Fig. 8. Outage probability for a DBF scheme with channel estimation error.

Fig. 9. Average capacity of a spectrum sharing system with user selection and optimal power adaptation.

Fig. 10. Outage probability for a DBF scheme with co-channel interferers.

\( N = 2, 3, 4 \). We can observe that the capacity evaluated by using the approximate cutoff value is tight at high SNR. Also, we can see that the capacity improvement due to power adaptation for large number of secondary users or high SNR is negligible.

In Fig.10, we plot the lower bound curves according to the analysis (58) for \( N = 2, 3 \) and \( L = 4 \). The total interference power \( P_I \) is set to 10 dB. \( P_{I,1} = 1, P_{I,2} = 7, P_{I,3} = 0.8, P_{I,4} = 1.2, \) and \( \gamma_{th} = 3 \) dB. For comparison, we also present the simulation results for the exact outage probability. As expected, the simulation results are in agreement with their corresponding analytical results.

**VII. CONCLUSIONS**

In this paper, we presented a performance analysis for a DBF system operating in a spectrum sharing network. Results show that the proposed scheme has a better performance than the user selection scheme investigated in [4]. The reason is that the optimal DBF with perfect feedback can maximize the received SNR at the secondary receiver. However, the optimal DBF is not practical in realistic wireless communications. Thus, we analyzed the RVQ scheme. We observed that the quantization loss can be made up by increasing the dimension of the codebook. Due to complexity of the performance analysis, in this paper, we apply some bounds. While, the approximate analytical results maybe do not always coincide with the simulation based-exact results, these closed-form expressions provide some explicit insights and can characterize the scaling law of the system performance.
The MGF of $\gamma M_\gamma(s)$ is readily shown to be given by

$$M_\gamma(s) = \prod_{i=1}^{N} M_{\gamma_i}(s),$$

where $M_{\gamma_i}(s)$ is the MGF of $\gamma_i$. With the aid of [22, Eq.(3.353.3)] and the PDF given in (5), $M_{\gamma_i}(s)$ can be expressed as

$$M_{\gamma_i}(s) = \frac{Q}{\rho^2} \int_{0}^{\infty} \frac{e^{-\gamma / 2}}{(Q \rho^{-2} + \gamma / 2)^2} d\gamma = 1 - \frac{Q}{\rho^2} se^{s2} E_1 \left( \frac{Q}{\rho^2} \right).$$

Hence, with the i.i.d assumption, the MGF for the sum $\gamma$ is simply given by

$$M_\gamma(s) = \left( 1 - \frac{Q}{\rho^2} se^{s2} E_1 \left( \frac{Q}{\rho^2} \right) \right)^N.$$  \hspace{1cm} (61)

With the MGF (61) in hand, we can evaluate the BER performance by using the well-known MGF-based method [31] as follows

$$P_b^{OBF}(E) = \frac{1}{\pi} \int_{0}^{\pi/2} M_\gamma \left( \frac{q}{\sin^2 \theta} \right) d\theta.$$  \hspace{1cm} (62)

Thus, substituting (61) into (62) results in (15). Normally, with the MGF, we can evaluate the PDF of a random variable by using the inverse Laplace Transform method. However, it is difficult to obtain the PDF of $\gamma$ in closed-form from (61). Therefore, we need to use some asymptotic analysis.

**APPENDIX II**

**PROOF OF LEMMA 2**

By using the approach developed in [18], we can decompose the equivalent channel $h$ into two parts in terms of $w$ and its corresponding orthogonal complement $w^\perp$. Using the result in [18], we have

$$Z = \frac{|hw|^2}{|h|^2} = \frac{|h_a|^2}{|h_a|^2 + |h_b|^2},$$

where $h_a$ is a scalar and $h_b$ is a vector with $N - 1$ elements. Unlike in [18], here $|h_a|^2$ and $Y = ||h_b||^2$ are not simple Gamma distributed RVs. From (5), the PDF of $V = |h_a|^2$ can be easily obtained. To obtain the CDF of $Z$, we have to find first the PDF of $Y$. In the following, we use the bounds (67)

$$\beta_{L} = \max_{1 \leq i \leq N-1} \left( \beta_i^2 / \alpha_i^2 \right) < Y < (N-1) \max_{1 \leq i \leq N-1} \left( \beta_i^2 / \alpha_i^2 \right)$$

Thus, from (7), we can obtain the PDFs of $Y_L$ and $Y_U$ as

$$f_{Y_L}(y) = (N-1) \rho^{2(N-1)} y^{N-2} (1 + \rho^2 y)^{-N}$$

and

$$f_{Y_U}(y) = (N-1) \rho^{2(N-1)} y^{N-2} (N-1 + \rho^2 y)^{-N}.$$  \hspace{1cm} (64)

From (63), the CDF of $Z$ is given by

$$F_Z(z) = \int_{0}^{\infty} F_V \left( \frac{2y}{1-z} \right) f_Y(y) dy.$$  \hspace{1cm} (66)

Substituting (64) and (65) into (66) and after some mathematical manipulations, the CDF of $Z$ can be bounded as

$$1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - 2z}{1 - z} \right) < F_Z(z) < 1 - \frac{1}{N^2} F_1 \left( N - 1, 1; N + 1; \frac{1 - N z}{1 - z} \right).$$  \hspace{1cm} (67)

To get (67), the integral table [22, Eq.(3.197.1)] and the equality [22, Eq.(9.131.1)] were used. Based on $F_1(a, b; c; x) \rightarrow 0$ when $x \rightarrow \infty$ and [22, Eq.(9.122.1)], we can verify that $0 \leq F_Z(z) \leq 1$ for $z \in (0, 1)$. Then, using the order statistics, we conclude the proof for Lemma 2.


