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ON THE MICROLOCAL REGULARITY OF THE ANALYTIC VECTORS  
FOR “SUMS OF SQUARES” OF VECTOR FIELDS

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This is joint work with M. Derridj. We prove via FBI-transform a result concerning the optimal microlocal Gevrey regularity of analytic vectors for operators sums of squares of vector fields with real-valued real analytic coefficients of Hörmander type, thus providing a microlocal version, in the analytic category, of a result due to M. Derridj (*“Local estimates for Hörmander’s operators of first kind with analytic Gevrey coefficients and application to the regularity of their Gevrey vectors”*, Pacific Journal of Mathematics 302 (2019), No. 2, 511–543.) concerning the problem of the local regularity for the Gevrey vectors for sums of squares of vector fields with real-valued real analytic/Gevrey coefficients.

The study of the regularity of analytic vectors of partial differential operators goes back to the work of T. Kotake and N. S. Narasimhan (*“Regularity theorems for fractional powers of a linear elliptic operator”*, Bull. Soc. Math. France 90 (1962), 449–471.) This property, called the iteration property was further studied in the following decades in more general situations such as systems, or non-elliptic operators. This was, in particular, the case for the class of differential operators of principal type with analytic coefficients and also, after the famous article by L. Hörmander on hypoelliptic operators of second order (*“Hypoelliptic second order differential equations”*, Acta Math. 119 (1967), 147171), the systems of real-analytic real vector fields satisfying the so-called Hörmander condition, and also for the Hörmanders operators themselves. In the paper we focus on operators of Hörmander type of first kind:

$$P(x, D) = \sum_{j=1}^m X_j^2(x, D),$$

where  $X_1(x, D), \dots, X_m(x, D)$  are vector fields with real-valued real analytic coefficients on  $U$ , open neighborhood of the origin in  $\mathbb{R}^n$ . We address the problem of the microlocal Gevrey regularity of the analytic vectors of  $P$ :  $u \in \mathcal{D}'(U)$  is an analytic vector for  $P$  if for any compact subset  $K$  of  $U$  there exists a positive constant  $C_K$  such that

$$\|P^N u\|_{L^2(K)} \leq C_K^{2N+1} (2N!)^s, \quad \forall N \in \mathbb{Z}_+.$$

We prove that the optimal index of the microlocal Gevrey regularity of the analytic vectors of  $P$  is related to the index of subellipticity of the operator.