VIRTUAL EAST-WEST SCV SEMINAR

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Local and global visibility and Gromov hyperbolicity for domains in $\mathbb{C}^n$

Consider, for simplicity, a bounded domain Ω in $\mathbb{C}^n$ endowed with Kobayashi metric, which is complete for the induced distance. As a geodesic metric space, it can be Gromov hyperbolic (meaning that every geodesic triangle must be δ-thin for some $\delta < \infty$). For $\mathcal{C}^1$-smooth convex domains, this is known to be equivalent to finite type. Any isometry from a Gromov-hyperbolic metric space can be extended to its (abstract) Gromov boundary. Another property that $(\Omega, \partial\Omega)$ can have is visibility, meaning that geodesics connecting points close to two distinct points of the boundary have to intersect some fixed compact set (depending on the boundary points). When Ω is Gromov-hyperbolic and its Gromov boundary coincides with $\partial\Omega$, it is visible. But there are many other cases of visibility.
We address the question of whether those two properties are local with respect to the boundary, that is, whether it is enough to verify them in $\Omega \cap U$, for U a neighborhood of $p \in \partial\Omega$, for every p. The main result is that if for each $p \in \partial\Omega$, there is a neighborhood U of p such that $\Omega \cap U$ is Gromov-hyperbolic and visible, then Ω enjoys both properties.
