A 2-dimensional vector is an "arrow" specified by a direction and a length (or magnitude).

Example: The vector of length $\sqrt{2}$ making an angle of $\frac{\pi}{4}$ from the positive $x$-axis (measured anticlockwise) can be graphically represented as

\[
\begin{align*}
\text{length} &= \sqrt{1^2 + 1^2} = \sqrt{2}.
\end{align*}
\]

or

Even though it's positioned differently, it's the same vector because it has the same length and direction.

Another way to specify a vector is by an ordered pair of numbers.

Example: The vector above is $\langle 1, 1 \rangle$

(run rise)

(Remember that this is different from $(1, 1)$, which denotes a point.)
Notation:

- We usually write \( \vec{a} = \langle a_1, a_2 \rangle \) for a vector. The numbers \( a_1, a_2 \) are called the components of \( \vec{a} \).

- The vector \( \vec{AB} \) is the vector which points from the point \( A \) to the point \( B \).

**Example:** If \( A = (1, 4) \), \( B = (3, 7) \)

\[
\vec{AB} = \langle 3-1, 7-4 \rangle = \langle 2, 3 \rangle
\]

In general, if \( A = (x_1, y_1) \), \( B = (x_2, y_2) \), then

\[\vec{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle\]

- The position vector of a point \( P \) is \( \vec{OP} \) where \( O \) is the origin.

**Example:** The position vector of \( (1,4) \) is \( \langle 1,4 \rangle \).

- The length of a vector \( \vec{a} \) is denoted \( |\vec{a}| \).

**Example:** Find the length of the vector \( \vec{a} = \langle 2, 3 \rangle \).

By Pythagoras it's \( |\vec{a}| = \sqrt{2^2 + 3^2} = \sqrt{13} \).

In general, if \( \vec{a} = \langle a_1, a_2 \rangle \) then \( |\vec{a}| = \sqrt{a_1^2 + a_2^2} \).

**Example about angles:**

Find the angle (measured anticlockwise) which the following vectors make with the positive \( x \)-axis:
(i) $\mathbf{a} = \langle 1, \sqrt{3} \rangle$

\[
\tan \theta = \frac{\sqrt{3}}{1} \quad \theta = \tan^{-1} \left( \frac{\sqrt{3}}{1} \right) = \frac{\pi}{3}.
\]

(ii) $\mathbf{a} = \langle -1, \sqrt{3} \rangle$

Wrong answer: $\tan^{-1} \left( \frac{\sqrt{3}}{-1} \right) = -\frac{\pi}{3}$

Right answer:

First draw a picture:

So $\theta = \pi - \frac{\pi}{3}$

A vector which has length 1 is called a unit vector.

Example: $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ is a unit vector.

Some unit vectors have special names:

$\mathbf{i} = \langle 1, 0 \rangle$  $\mathbf{j} = \langle 0, 1 \rangle$
Vector addition

If \( \vec{a} = \langle a_1, a_2 \rangle \) , \( \vec{b} = \langle b_1, b_2 \rangle \)
then \( \vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \) (add component wise)

Graphically,

What does \( \vec{a} + \vec{b} \) look like?

Scalar Multiplication

If \( c \) is a scalar (a real number) and \( \vec{a} = \langle a_1, a_2 \rangle \) a vector, then
\[
C\vec{a} = \langle ca_1, ca_2 \rangle
\]

So for example,
\[
3.5 \vec{a} = \langle 3.5a_1, 3.5a_2 \rangle
\]
It points in the same direction as \( \vec{a} \) but is three and a half times as long. That's because...
\[ |3.5 \vec{a}| = \sqrt{(3.5 a_1)^2 + (3.5 a_2)^2} \]
\[ = \sqrt{(3.5)^2 (a_1^2 + a_2^2)} \]
\[ = 3.5 \sqrt{a_1^2 + a_2^2} \]
\[ = 3.5 |\vec{a}|. \]

In general, \[ |c \vec{a}| = |c| |\vec{a}|. \]

- \( c \vec{a} \) points in the same direction as \( \vec{a} \) if \( c > 0 \).
- \( c \vec{a} \) points in the opposite direction of \( \vec{a} \) if \( c < 0 \).

Note that \( 5\vec{a} \) is the same as \( \vec{a} + \vec{a} + \vec{a} + \vec{a} + \vec{a} \).

**Example:** Find a unit vector which points in the same direction as the vector \( 3\vec{i} + 4\vec{j} \).

\[ 3\vec{i} + 4\vec{j} = \langle 3,4 \rangle. \] This vector has length \[ \sqrt{3^2 + 4^2} = 5. \]

To get a unit vector, we should scale \( \langle 3,4 \rangle \) down by a factor of 5.

**Answer:** \[ \frac{1}{5} \cdot \langle 3,4 \rangle = \langle \frac{3}{5}, \frac{4}{5} \rangle \]

(Check that \( \langle \frac{3}{5}, \frac{4}{5} \rangle \) does indeed have length 1)

**Vector subtraction**

\[ \vec{a} = \langle a_1, a_2 \rangle, \quad \vec{b} = \langle b_1, b_2 \rangle \]
\[ \vec{a} - \vec{b} \text{ is the same as adding } \vec{a} \text{ and } -\vec{b} \]
\[ \vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle. \]
Applications

1. A velocity is a vector. The magnitude of velocity is speed.
   A ship is required to travel south at 20 km/h.
   However, there is a constant water current of 6 km/h from the south west.
   Find the direction and speed the ship must travel to compensate for the current.

   \[ \vec{w} = \langle 6 \cos \frac{\pi}{4}, 6 \sin \frac{\pi}{4} \rangle = \langle 6/\sqrt{2}, 6/\sqrt{2} \rangle. \]

   \[ \begin{align*}
   \cos \frac{\pi}{4} &= \frac{w_1}{6} \Rightarrow w_1 = 6 \cos \frac{\pi}{4} \\
   \sin \frac{\pi}{4} &= \frac{w_2}{6} \Rightarrow w_2 = 6 \sin \frac{\pi}{4} 
   \end{align*} \]

   Let \( \vec{r} = \langle 0, -20 \rangle \) (the required resultant velocity).
   Let \( \vec{s} \) be the velocity with which the ship must actually travel.
We have the equation 
\[ \vec{r} = \vec{s} + \vec{w} \]

\[ \langle 0, -20 \rangle = \langle s_1, s_2 \rangle + \langle \frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}} \rangle \]

\[ \langle 0, -20 \rangle = \langle s_1 + \frac{6}{\sqrt{2}}, s_2 + \frac{6}{\sqrt{2}} \rangle \]

\[ 0 = s_1 + \frac{6}{\sqrt{2}} \Rightarrow s_1 = -\frac{6}{\sqrt{2}} \]

\[ -20 = s_2 + \frac{6}{\sqrt{2}} \Rightarrow s_2 = -20 - \frac{6}{\sqrt{2}} \]

\[ \therefore \text{ ship should travel with velocity } \langle -\frac{6}{\sqrt{2}}, -20 - \frac{6}{\sqrt{2}} \rangle \]

\[ \text{speed} = \sqrt{\left( \frac{6}{\sqrt{2}} \right)^2 + \left( -20 - \frac{6}{\sqrt{2}} \right)^2} \text{ km/h} \]

\[ \theta = \tan^{-1} \left( \frac{\frac{6}{\sqrt{2}}}{20 + \frac{6}{\sqrt{2}}} \right) \]

2. A force is a vector.

A 100 lb weight hangs from two wires as shown.

Find the size of the tension in each wire (in lb).
All forces must cancel out in an equilibrium, so

\[ \vec{T}_1 + \vec{T}_2 + \langle 0, -100 \rangle = \vec{0} \]

\[ \vec{T}_1 = \langle |\vec{T}_1| \cos 50, |\vec{T}_1| \sin 50 \rangle \]

\[ \vec{T}_2 = \langle |\vec{T}_2| \cos 32, |\vec{T}_2| \sin 32 \rangle \]

\[ \langle 0, 0 \rangle = \langle -|\vec{T}_1| \cos 50, |\vec{T}_1| \sin 50 \rangle + \langle |\vec{T}_2| \cos 32, |\vec{T}_2| \sin 32 \rangle + \langle 0, -100 \rangle \]

**x-components** add up to zero:

\[-|\vec{T}_1| \cos 50 + |\vec{T}_2| \cos 32 = 0\]

**y-components** add up to zero:

\[|\vec{T}_1| \sin 50 + |\vec{T}_2| \sin 32 - 100 = 0\]

Two equations in two unknowns (\( |\vec{T}_1| \) and \( |\vec{T}_2| \)).

You should be able to solve it.

and you should get (using a calculator):

\[ |\vec{T}_1| \approx 85.64 \text{ lb} \]

\[ |\vec{T}_2| \approx 64.91 \text{ lb} \]
Section 1.2

How do we multiply together two vectors?

One way, inspired by Physics, is the dot product.

Definition

A. In terms of lengths and angles of vectors:

If \( \vec{a} \) and \( \vec{b} \) are two vectors and \( \theta \) is the angle between them (\( 0 \leq \theta \leq \pi \)) then

\[
\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta
\]

B. In terms of components of vectors:

If \( \vec{a} = \langle a_1, a_2 \rangle \), \( \vec{b} = \langle b_1, b_2 \rangle \), then

\[
\vec{a} \cdot \vec{b} = \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1b_1 + a_2b_2
\]

The two definitions are equivalent. They would give you the same answer.

Remember: vector \cdot vector = number

Example: Find the angle between the two vectors

\( \vec{a} = \langle 2, 2 \rangle \) and \( \vec{b} = \langle 5, -3 \rangle \).

On the one hand,

\[
\vec{a} \cdot \vec{b} = 2 \cdot 5 + 2 \cdot (-3) = 4
\]
On the other hand,
\[ \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta = \sqrt{2^2 + 2^2} \sqrt{5^2 + 3^2} \cos \theta = \sqrt{8 \cdot 134} \cos \theta \]

So \( \sqrt{8 \cdot 134} \cos \theta = 4 \)

\[ \cos \theta = \frac{4}{\sqrt{8 \cdot 134}} \]

\[ \theta = \cos^{-1} \left( \frac{4}{\sqrt{8 \cdot 134}} \right) \]

**Application: Work**

1. A wagon is pulled a distance of 100 m along a horizontal path by a constant force of 50 N. The handle of the wagon is at an angle of 30° above the horizontal. How much work is done?

![Diagram of force and displacement vectors](image)

From Physics, Work = Size of force \times Distance traveled in the direction of motion

In this problem, Distance traveled = 100 m.

We need to find the force in the direction of \( \mathbf{F} \) (also called the component of \( \mathbf{F} \) along \( \mathbf{B} \) or the scalar projection)
of \( \vec{F} \) onto \( \vec{B} \). In this problem, it's the horizontal component of \( \vec{F} \) which we're after.

\[
\cos 30 = \frac{\text{adjacent}}{50}
\]

\[\therefore \text{adjacent} = 50 \cos 30\]

\[\therefore \text{Work} = 50 \cdot \cos 30 \cdot 100 \text{ J}\]

We have also found a formula:

\[
\text{Work} = |\vec{F}| \cdot \cos \theta \cdot |\vec{D}| \tag{angle between \( \vec{F} \) and \( \vec{D} \)}
\]

\[
\text{Work} = \vec{F} \cdot \vec{D}
\]

"looks" just like work = force times distance

2. A boat sails in a straight line from the point (2,1) to the point (5,3) with the help of the wind, which provides a constant force with vector representation \( \vec{F} = 4\hat{i} + 3\hat{j} \). Find the work done by the wind, if distance is measured in m and force in N.

\[
\vec{F} = <4, 37>
\]

\[
\vec{D} = <5-2, 3-1> = <3, 27>
\]

\[\therefore \text{Work} = <4, 37> \cdot <3, 27> = 18 \text{ J}\]
Projection

In the figure, \( \vec{c} \) is the vector projection of \( \vec{b} \) onto \( \vec{a} \) and it is denoted \( \text{proj}_{\vec{a}} \vec{b} \). (Think about it like this. If \( \vec{b} \) is a force and \( \vec{a} \) the displacement vector, then \( \vec{c} = \text{proj}_{\vec{a}} \vec{b} \) is the force which acts in the direction of motion and contributes to work.)

The length of \( \vec{c} = \text{proj}_{\vec{a}} \vec{b} \) is called the **scalar projection** of \( \vec{b} \) onto \( \vec{a} \) or the **component** of \( \vec{b} \) along \( \vec{a} \) and it is denoted \( \text{comp}_{\vec{a}} \vec{b} \). (Think about it as the size of the force in the direction of motion.)

**Note:** When \( \cos \theta < 0 \), \( \text{comp}_{\vec{a}} \vec{b} \) is actually negative of the length of \( \text{proj}_{\vec{a}} \vec{b} \).

By trigonometry,

\[
|\vec{c}| = |\vec{b}| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}
\]

\[
\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}
\]

So now we know the length of \( \vec{c} \). If we multiply this by a unit vector in the direction of \( \vec{a} \), then we'll get \( \vec{c} \):

\[
\vec{c} = |\vec{c}| \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \frac{\vec{a}}{|\vec{a}|}
\]

\[
\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}
\]
Example:

Find the distance from the point \((3, 1)\) to the line \(y = 1 - x\).

We need to find \(a\).

We see that \(b = \text{component of } \langle 3, 0 \rangle \text{ along } \langle 1, -1 \rangle\)

\[
= \frac{\langle 3, 0 \rangle \cdot \langle 1, -1 \rangle}{|\langle 1, -1 \rangle|} = \frac{3}{\sqrt{2}}
\]

So \(a = \sqrt{3^2 - b^2} = \sqrt{3^2 - \left(\frac{3}{\sqrt{2}}\right)^2} = \frac{3}{\sqrt{2}}\).
Final notes:

1. \( \vec{a} \cdot \vec{a} = <a_1, a_2> \cdot <a_1, a_2> = a_1^2 + a_2^2 \)

So \( \sqrt{\vec{a} \cdot \vec{a}} = |\vec{a}| \).

2. In many ways, the dot product behaves like normal multiplication. i.e. it has similar properties:
   \( \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \)
   \( \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \)

3. Two non-zero vectors \( \vec{a}, \vec{b} \) are called orthogonal or perpendicular if the angle between them is \( \theta = \frac{\pi}{2} \).

![Diagram of vectors \( \vec{a} \) and \( \vec{b} \) forming a right angle.]

\( \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \)

\[ \therefore \ \vec{a}, \vec{b} \text{ orthogonal } \iff \vec{a} \cdot \vec{b} = 0 \]

Example: Find a vector which is perpendicular to \( <2,4> \).

We need to find a vector \( \vec{a} = <a_1, a_2> \) such that

\( <a_1, a_2>, <2, 4> = 2a_1 + 4a_2 = 0 \).

Choose any non-zero value for \( a_1 \), then solve for \( a_2 \).
If \( a_1 = 1 \), \( 2 \cdot 1 + 4a_2 = 0 \Rightarrow a_2 = -\frac{1}{2} \). So \( \vec{a} = <1, -\frac{1}{2}> \).
Section 1.3

Imagine a bug moving around on the x-y plane. Imagine that we trace out its path from $t=0$ to $t=100$ seconds, and it looks like

The bug's position at time $t$ depends on $t$. So its coordinates are given by $(x(t), y(t))$ for some functions $x(t)$ and $y(t)$. The vector $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ is the position vector of the bug at time $t$. It's a vector function of $t$. The bug's path is completely known if $\mathbf{r}(t)$ is specified.

Examples:

1. A cannonball is fired from a cannon. At time $t$ secs after being fired, the cannonball's position is given by $(x(t), y(t))$, where

$$x(t) = 500 \, t$$
$$y(t) = 1000 \, t - 16t^2$$

These are called the parametric equations describing the path of the cannonball. The parameter is $t$.

(Distance measured in feet)
The path of the cannonball can also be described by the vector equation \( \mathbf{r}(t) = \langle 500t, 1000t - 16t^2 \rangle \).

The path can also be described by a Cartesian equation as follows. A Cartesian equation involves only \( x \) and \( y \). So we should eliminate \( t \).

\[
x = 500t \implies t = \frac{x}{500}
\]

\[
y = 1000t - 16t^2 = 2x - \frac{16}{250,000} x^2.
\]

The Cartesian equation is \( y = 2x - \frac{16}{250,000} x^2 \)

\[
y = x \left( 2 - \frac{16}{250,000} x \right),
\]

which is a parabola that looks like:

2. The vector function \( \mathbf{r}(\theta) = \langle 3 \cos \theta, 3 \sin \theta \rangle \) for \( 0 \leq \theta < 2\pi \) describes a circle. Why?

Parametric equations: \( x(\theta) = 3 \cos \theta \)
\[
y(\theta) = 3 \sin \theta.
\]

So \( x^2 + y^2 = 9 \cos^2 \theta + 9 \sin^2 \theta = 9(\cos^2 \theta + \sin^2 \theta) = 9. \)
Cartesian equation: \( x^2 + y^2 = 9 \).

This a circle with radius \( r = 3 \), center \( (0,0) \).

As \( \theta \) goes from \( 0 \) to \( 2\pi \),
one full revolution of the circle
is completed.

3. Describe the curve given by the vector equation
\[
\vec{r}(t) = \langle \cos(2t) + 1, \sin(2t) \rangle
\]
for \( 0 \leq t < \frac{\pi}{2} \).

Parametric:
\[
x(t) = \cos(2t) + 1, \quad y(t) = \sin(2t)
\]

Cartesian:
\[
\cos(2t) = x(t) - 1
\]

So \((x-1)^2 + y^2 = \cos^2(2t) + \sin^2(2t) = 1\)

\((x-1)^2 + y^2 = 1\) is the equation of a circle with
center \((1,0)\) and radius 1.

But since \( t \) only goes from \( 0 \) to \( \frac{\pi}{2} \), \( 2t \) goes from
\( 0 \) to \( \pi \), and so only half of a circle is traced out.

\[
\vec{r}(0) = \langle 2, 0 \rangle
\]
\[
\vec{r}(\frac{\pi}{2}) = \langle 0, 0 \rangle
\]

4. A curve is described by the parametric equations
\[
\begin{align*}
\begin{array}{l}
x(t) = t^2 - 2t \\
y(t) = t + 1
\end{array}
\end{align*}
\]
for \( t \in (-\infty, \infty) \).

(a) For which values of \( t \) does the curve cross the
y-axis?

(b) Sketch the curve.

(a) We need to solve \( x(t) = 0 \)
\[
t^2 - 2t = t(t - 2) = 0 \Rightarrow t = 0, 2.
\]
(b) Convert to Cartesian form. Eliminate $t$ first.

$x = t^2 - 2t \Rightarrow$ difficult to solve for $t$

$y = t + 1 \Rightarrow$ easy to solve for $t$: $t = y - 1$

So $x = t^2 - 2t = (y-1)^2 - 2(y-1) = y^2 - 4y + 3$

$x = y^2 - 4y + 3$ is a parabola "on its side" (switch $x$ and $y$ in your mind)

5. The path of an object is described by

$x(t) = (t-1)(t-2)$

$y(t) = \sin(\pi t)$

where $t$ is time. Show that the path of the object crosses itself at the origin.

What is the question asking? We have to show that the object visits the origin more than once. I.e. show that \( \{ x(t) = 0 \} \) has more than one solution.

\[
\begin{align*}
x(t) &= (t-1)(t-2) = 0 \Rightarrow t = 1, 2 \\
y(t) &= \sin(\pi t) = 0 \Rightarrow t = \text{any integer} \{ \text{common set: } t = 1, 2 \}
\end{align*}
\]

The object passes through the origin twice: at $t = 1$ and $t = 2$. 
Vector Equation of a Line

1. Find the vector equation of the line which passes through \((0,0)\) and is parallel to \(\langle 1, 2 \rangle\).

\[
\vec{r}(t) = t \langle 1, 2 \rangle, \quad t \in (-\infty, \infty)
\]

Why? Because multiplying a vector by a scalar \(t\) stretches or shrinks it, in the same direction if \(t > 0\) and opposite direction if \(t < 0\). So we can get anywhere on the line by multiplying \(\langle 1, 2 \rangle\) by some \(t \in (-\infty, \infty)\).

2. Find the vector equation of the line which passes through \((1,0)\) and is parallel to \(\langle 1, 2 \rangle\).
This time, the equation is
\[ \vec{r}(t) = \langle 0,1 \rangle + t \langle 1,2 \rangle \]
\[ t \in (-\infty, \infty) \]

Second, go anywhere along the line by adding \( t \langle 1,2 \rangle \)

First go to \( (0,1) \) by adding \( \langle 0,1 \rangle \)

\[ \vec{r}(t) = \langle 0,1 \rangle + t \langle 1,2 \rangle \]
(This is the position vector of the points on the line)

3. In general, the equation of the line which passes through \( (x_0, y_0) \) and is parallel to \( \langle v_1, v_2 \rangle \) is given by

Vector equation:
\[ \vec{r}(t) = \langle x_0, y_0 \rangle + t \langle v_1, v_2 \rangle \]
\[ t \in (-\infty, \infty) \]

\[ \vec{r}(t) = \langle x_0 + tv_1, y_0 + tv_2 \rangle \]

Parametric equations:
\[ x(t) = x_0 + tv_1 \]
\[ y(t) = y_0 + tv_2 \]

Notice both are linear in \( t \)
The slope of the line is \[ \text{slope} = \frac{v_2}{v_1} \text{ if } v_1 \neq 0 \]

Example: Find the distance from the point \((3, 1)\) to the line \(L: y = 1-x\).

\[ L: y = 1-x \]

\[ \text{line } M \]

\[ (3, 1) \]

\[ \text{The line } M \text{ which passes through } (3, 1) \text{ and is parallel to } \langle 1, 1 \rangle \]

\[ \text{has vector equation } \quad \vec{r}(t) = \langle 3, 1 \rangle + t \langle 1, 1 \rangle. \]

\[ \vec{r}(t) = \langle 3 + t, 1 + t \rangle \]

\[ \text{The line } M \text{ crosses the line } L \text{ when} \]

\[ y(t) = 1 - x(t) \]

\[ 1 + t = 1 - (3 + t) \quad \Rightarrow \quad t = -\frac{3}{2}. \]

So they cross at \((x(-\frac{3}{2}), y(-\frac{3}{2})) = (3 - \frac{3}{2}, 1 - \frac{3}{2}) = (\frac{3}{2}, -\frac{1}{2})\).
The distance between \((3,1)\) and \((3\tfrac{1}{2}, -\tfrac{1}{2})\) is
\[
\sqrt{(3-3\tfrac{1}{2})^2 + (1-(-\tfrac{1}{2}))^2} = \frac{3}{\sqrt{2}}
\]

Note: see pg. 60 for a different way to do this problem.

Example: Two lines \(L_1\) and \(L_2\) are given by the following vector equations:

\[
L_1: \vec{r}_1(t) = \langle 1 + 2t, 2 + 3t \rangle
\]
\[
L_2: \vec{r}_2(t) = \langle 8 - t, 10 + t \rangle
\]

At which point (if any) do they intersect?

Solution: Call the point of intersection \((a,b)\).

- Suppose that \(L_1\) is at the point \((a,b)\) at "time" \(t_1: \langle 1 + 2t_1, 2 + 3t_1 \rangle = \langle a, b \rangle\)

- Suppose that \(L_2\) is at the point \((a,b)\) at "time" \(t_2: \langle 8 - t_2, 10 + t_2 \rangle = \langle a, b \rangle\)

\[
\begin{align*}
1 + 2t_1 &= 8 - t_2 \\
2 + 3t_1 &= 10 + t_2
\end{align*}
\]
\[
\Rightarrow t_1 = 3, \ t_2 = 1
\]

So \(\langle a, b \rangle = \langle 1 + 2 \cdot 3, 2 + 3 \cdot 3 \rangle = \langle 7, 11 \rangle\)

\((a,b) = (7,11)\)